Reg. No._____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FOURTH SEMESTER B.TECH DEGREE EXAMINATION, MAY 2017

Course Code**: MA204**

Course Name**: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS (AE, EC)**

Max. Marks: 100 Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A

Answer any two questions.

1(a) The probability distribution of a discrete random variable X is given by $P(X=x)=\frac{k}{2^x}$, $x=0,1,2,3,4$. Find (i) the value of k,(ii) the probability that X is even and (iii) E(X). (7)

(b) The probability that an electric component manufactured by a firm is defective is 0.01. If the produced items are sent to the market in packets of 10, find the number of packets containing exactly two defectives and at most two defectives in a consignment of 1000 packets using (i) binomial distributionand (ii) Poisson approximation to binomial distribution. (8)

2(a) Buses arrived at a specified stop at 15 minute intervals starting at 7 AM. A passenger arrives at the stop at random time between 7 AM and 7.30 AM. Find the probability that he waits (i) less than 5 minutes, (ii) at least 12 minutes. (7)

(b) 1000 light bulbs with mean length of life 120 days are installed in a factory. Their length of life is assumed to follow normal distribution with S.D 20 days. How many bulbs will expire in less than 90 days? If it is decided to replace all the bulbs together, what interval should be allowed between replacements if not more than 10% should expire before replacement? (8)

3(a) A communication system sends data in the form of packets of fixed length. Noise in the communication channel may cause a packet to be received incorrectly. If this happens, the packet is retransmitted. Let the probability that a packet is received incorrectly is p. Determine the average number of transmissions that are necessary before a packet is received correctly. (7)

(b) Suppose a new machine is put into operation at time zero. Its life time is an exponential random variable with mean life 12 hours. (i) What is the probability that the machine will work continuously for one day? (ii) Suppose the machine has not failed by the end of the first day, what is the probability that it will work for the whole of the next day? (8)

PART B

Answer any two questions.

4(a) A computer generates 100 random numbers which are uniformly distributed between 0 and 1. Find approximately the probability that their sum is at least 50. (7)

(b) The joint distribution of a two-dimensional random variable (X,Y) is given by p(x,y)=c(2x+3y), x=0,1,2 ; y=1,2,3. Find (i) the value of c (ii) the marginal distributions (iii) Are X and Y independent? (8)

5(a) Prove that the random process X(t) is defined by $X(t)=a\sin(\omega t+\theta)$, where a and ω are constants and θ is a random variable uniformly distributed in $[0,2\pi]$ is a WSS process. (7)

(b) In each of the following examine whether $f(\omega)$ could be the power spectral density(PSD) of a wide sense stationary process. Explain your reasoning.

$$(i)\ f(\omega)=\begin{cases}\dfrac{\sin\omega}{\omega}, & \omega\neq 0\\ 0, & \omega=0\end{cases} \qquad (ii)\ f(\omega)=\begin{cases}\pi, & |\omega|<1\\ 0, & otherwise\end{cases}$$

If $f(\omega)$ is a valid PSD find the corresponding autocorrelationfunction. (8)

6(a) Let X_i are independent random variables taking values -1 and 1 with probability $\frac{1}{2}$.

A random process Z_n is defined as $Z_n=X_1+X_2+...+X_n$, $n=1,2,.....$ Is the process a WSS process? (7)

(b) A pair of random variables X and Y have a joint probability density function given by

$$f(x,y)=\begin{cases}\dfrac{1}{\pi}, & x^2+y^2\le 1\\ 0, & otherwise\end{cases}$$

. Show that X and Y are not independent, but uncorrelated. (8)

PART C

Answer any two questions.

7(a) Obtain the probability distribution of the time between two consecutive occurrences of a Poisson process. (4)

(b) The arrival of patients at a doctor's consulting room is found to follow a Poisson process with an average of one in 5 minutes. The room can accommodate a maximum of 4 persons and if more people come, they wait outside the room. If patients start coming from 8 A.M. onwards, (i) What is the probability that the room is full when thedoctor arrives at 9 A. M.? (ii) If the doctor takes a break from 11A.M. to 11.15 A.M., and a lunch break from 1 P.M to 1.30 P.M. what is the probability that no new patients arrive during both the tea break and lunch break? (8)

(c) The transition probability matrix P of a Markov Chain with three states 1, 2 and 3 is

$$\begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

with initial distribution $P(X_0=1)= P(X_0=2)= P(X_0=3) = \frac{1}{3}$. Find $P(X_2=3)$ and $P(X_2=3, X_1=2, X_0=3)$. (8)

8(a) Using Newton-Raphson method, compute a real root of $e^{2x} - x - 6 = 0$ lying between 0 and 1. (4)

(b) Health surveys are conducted in a city every 10 years. The following data gives the number of people (in thousands) having heart diseases as found from the records of the survey.

Year	1961	1971	1981	1991	2001	2011
No. of people	16	19	23	28	34	41

Use Newton's interpolation method to estimate the number of people with heart diseases in the year 2005. (8)

(c) Using Runge-Kutta method of order four, compute y(0.2) given that $\frac{dy}{dx} = e^x + y$, y(0) = 0. Take step size h = 0.1 (8)

9(a) A meteorologist studying the weather in a region decides to classify each day as simply sunny or cloudy. After analysing several years of weather records, he finds that the day after a sunny day is sunny 80% of the time, and cloudy 20% of the time; and the day after a cloudy day is sunny 60% of the time, and cloudy 40% of the time. (i) If Monday is observed to be cloudy what is the probability that Wednesday is sunny? (ii) If the probability is 0.3 for Monday to be cloudy what is the probability that Wednesday is sunny? (iii) What is the probability for weather to be cloudy on Monday, cloudy again on Tuesday, then sunny on Wednesday and again sunny on Thursday in that order, given that the probability is 0.3 for Monday to be cloudy? (iv) What is the long term probability distribution of weather on any day? (10)

(b) The speed of a moving particle was measured at different points of time. The time t when the first measurement was recorded is taken as t =0. Subsequent speeds at different times are as shown in the following table.

Tim(t) in seconds	0	10	20	30	40	50	60
Velocity(v) in m/sec	35	39	44	50	56	43	40

Using Simpson's one-third method, evaluate the distance travelled by the particle in 60 seconds. (5)

(c) Using Lagrange's interpolation method find the polynomial f(x) which agree with the data f(-1) = 3, f(0) = -4, f(1) = 5 and f(2) = -6 (5)

Total Pages: 3

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FOURTH SEMESTER B.TECH DEGREE EXAMINATION, JULY 2017

Course Code: MA204

Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS (AE, EC)

Max. Marks: 100 Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A

Answer any two questions. Each carries 15 marks

1 a) A random variable X takes values 0,1, 2 and 3 with probabilities (7)

$$P(X=0)=\frac{8}{15},\ P(X=1)=\frac{1}{3},\qquad P(X=2)=P(X=3)=\frac{1}{15}$$

(i) Find the mean and variance of X.

If $Y=1000+300X$ find $P(Y\geq 1500)$ and $E[Y]$

b) In an examination, a candidate has to answer 15 multiple choice questions each of which has 4 choices for the answer. He knows the correct answer to 10 questions and for the remaining 5 questions he chooses the answer randomly. (8)

(i) What is the probability that he answers 13 or more questions correctly?

(ii) What is the mean and variance of the number of correct answers he gives?

2 a) The lifetime of a battery is exponentially distributed. 40% of such batteries do not last longer than 1000 hours. Mr. Kumar purchased such a battery which is already used for 500 hours. What is the probability that it will last another 1000 hours? (5)

b) Find the mean and variance of a random variable X which is uniformly distributed in the interval $[a,b]$ (5)

(c) The monthly salary (in Rs.) of 1000 employees in a factory are normally distributed with mean 20,000 and standard deviation 5000. Estimate the number of employees whose monthly salary will be (i) between 18,000 and 22,000 (ii) less than 18,000? (5)

3 a) Accidents occur at an intersection at a Poisson rate of 2 per day. (7)

(i) What is the probability that there would be no accidents on a given day?

(ii) What is the probability that in January there are at least 3 days (not necessarily consecutive) without any accidents?

b) A printer ink cartridge has a life of X hours under normal usage. The variable X is modelled by the probability density function (8)

$$f(x)=\begin{cases}\frac{k}{x^2}, & x\geq 400,\\ 0, & \text{otherwise.}\end{cases}$$

(i) Find k

(ii) Find the probability that such a cartridge has a life of at least 600 hours of normal usage.

(iii) Find the probability that two cartridges will have to be replaced before each has been used for 600 hours.

PART B

Answer any two questions. Each carries 15 marks

4 a) A factory has two outlets to sell its products. The daily sales from the first outlet is uniformly distributed between Rs. 50,000 and 60,000 and from the second outlet is uniformly distributed between 40,000 and 60,000. The sales of the outlets are independent. (7)

(i) What is the probability that the total sales from both the outlets combined is more than Rs.100000.

(ii) If 20% of the amount from the sales is profit, find the expected daily profit from both the outlets combined, and the variance of the profit.

b) A computer generates 100 random numbers uniformly distributed between 0 and 1. Use central limit theorem to find the probability that (8)

i) their sum is 60 or more,

ii) their average is 0.7 or less.

5 a) A random process $X(t)$ is defined by $X(t) = \sin(t + \Theta)$where Θ is a random variable taking values 0 or π with equal probability. Find the mean, autocorrelation and autocovariance of $X(t)$. Is it a wide sense stationary process? (7)

b) Find the power spectral density of a wide sense stationary process $X(t)$ with autocorrelation function $R_X(\tau) = e^{-3|\tau|}$. (8)

6 a) The joint probability distribution of two discrete random variables X and Y is given by (7)

$$p(x, y) = \frac{1}{30}(x + y), x = 0,1,2 \quad y = 0,1,2,3$$

Find the correlation coefficient between $X$and Y.

b) Find the autocorrelation function and average power of a wide sense stationary process $X(t)$ with power spectral density given by (8)

$$S_X(\omega) = \begin{cases} 1 - \omega, & |\omega| \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

PART C

Answer any two questions. Each carries 20 marks

7 a) The number of enquiries arriving at a call centre is a Poisson process with rate 5 per hour. (10)

i) Find the probability that there would be 3 calls between 10 AM and 11 AM and 4 calls between 2 PM and 4 PM.

ii) A call is categorized as `long' if it lasts more than 10 minutes. The probability that an arriving call is long is 0.2. Find the probability that the time between two consecutive long calls is less than 1 hour.

b) $X_n, n = 0,1,2, \ldots$is a Markov chain on state space $\{1,2,3\}$ with initial probability distribution $P(X_0 = 1) = P(X_0 = 2) = P(X_0 = 3) = 1/3$ and transition probability matrix given by (10)

$$\begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0.5 & 0.2 & 0.3 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

Find i) $P(X_2 = 3)$ ii) $P(X_1 = 3, X_2 = 2, X_3 = 2)$ iii) $P(X_2 = 1|X_0 = 2)$

8 a) The table gives the area under the normal probability curve from 0 to certain values x. (10)

x	0.5	1.0	1.5	2.0	2.5
$f(x)$	0.1905	0.3413	0.4332	0.4772	0.4938

Find $f(0.7)$ using suitable form of Newton's interpolation formula.

b) Use Runge-Kutta fourth order method to find $y(0.2)$and $y(0.4)$, given the initial value problem $\frac{dy}{dx} = e^x + y,\ y(0) = 0$ (10)

9 a) Find the probability distribution of the time between two consecutive arrivals in a Poisson process. (5)

b) A machine is in one of two states (i) down (state 0) or (ii) up (state 1). The transition probabilities between the states is given by the following matrix (TPM) (5)

$$\begin{bmatrix} 0.75 & 0.25 \\ 0.40 & 0.60 \end{bmatrix}$$

Find the proportion of time the machine will be up in the long run.

c) Find the real root of $f(x) = e^{2x} - x - 6$ lying between 0 and 1 using Newton-Raphson method. (5)

d) Evaluate $\int_0^1 e^{-x^2/2}\, dx$ numerically using Simpson's rule with a suitable step-size. (5)

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FOURTH SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

Course Code: MA204

Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS (AE, EC)

Max. Marks: 100 Duration: 3 Hours

(Normal distribution table is allowed in the examination hall)

PART A

Answer any two full questions, each carries 15 marks Marks

1 a) A random variable X has the following probability distribution: (7)

x	-2	-1	0	1	2	3
f(x)	0.1	k	0.2	2k	0.3	3k

Find: i) The value of k ii) Evaluate $P(X < 2)$ and $P(-2 < X < 2)$

iii) Evaluate the mean of X

b) The probability that a component is acceptable is 0.93. Ten components are picked at random. What is the probability that: (8)

i) At least nine are acceptable ii) At most three are acceptable.

2 a) Suppose that the length of a phone call in minutes is an exponential random variable with parameter $\lambda = \frac{1}{10}$. If someone arrives immediately ahead of you at a public telephone booth, find the probability that you will have to wait: (7)

i) More than 10 minutes ii) Between 10 and 20 minutes.

b) For a normally distributed population, 7% of items have their values less than 35 and 89% have their values less than 63. Find the mean and standard deviation of the distribution. (8)

3 a) Fit a binomial distribution to the following data and calculate the theoretical frequencies. (8)

x	0	1	2	3	4	5	6	7	8
f	2	7	13	15	25	16	11	8	3

b) The time between breakdowns of a particular machine follows an exponential distribution, with a mean of 17 days. Calculate the probability that a machine breaks down in a 15 day period. (7)

PART B

Answer any two full questions, each carries 15 marks

4 a) The joint PDF of two continuous random variables X and Y is given by (7)

$$f(x,y) = \begin{cases} kxy & 0 < x < 4,\ 1 < y < 5 \\ 0 & otherwise \end{cases}.$$

Find: i) k ii) The marginal distributions of X and Y

iii) Check whether X and y are independent.

b) A distribution with unknown mean μ has variance equal to 1.5. Use Central Limit Theorem to find how large a sample should be taken from the distribution in order that the probability will be at least 0.95 that the sample mean will be within 0.5 of the population mean. (8)

5 a) The autocorrelation function for a stationary process X(t) is given by $R_{XX}(\tau) = 9 + 2e^{-|\tau|}$. Find the mean value of the random variable $Y = \int_{\tau=0}^{2} X(t)dt$ and the variance of X(t). (7)

b) A random process X(t) is defined by $X(t) = Y(t)\cos(\omega t + \theta)$ Where Y(t) is a WSS process, ω is a constant and $\boldsymbol{\theta}$ is a random variable which is uniformly distributed in [0,2π] and is independent of Y(t). Show that X(t) is WSS. (8)

6 a) Consider the random process $X(t) = A\cos(\omega t + \theta)$ where A and ω are constants and $\boldsymbol{\theta}$ is a uniformly distributed random variable in (0,2π). Check whether or not the process is WSS. (7)

b) The joint PDF of two continuous random variables X and Y is (8)

$$f(x,y) = \begin{cases} 8xy\,, 0 < y < x < 1 \\ 0\,, \quad otherwise \end{cases}$$

i) Check whether X and Y are independent ii) Find $P(X + Y < 1)$

PART C

Answer any two full questions, each carries 20 marks

7 a) The number of particles emitted by a radioactive source is Poisson distributed. The source emits particles at a rate of 6 per minute. Each emitted particle has a probability of 0.7 of being counted. Find the probability that 11 particles are counted in 4 minutes. (4)

b) Assume that a computer system is in any one of the three states: busy, idle and under repair, respectively, denoted by 0,1,2. Observing its state at 2 P. M. each day, the transition probability matrix is $P = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{bmatrix}$ (8)

Find out the third step transition probability matrix and determine the limiting probabilities.

c) If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between two consecutive arrivals is: (8)

i) More than 1 minute ii) Between 1 minute and 2 minutes

iii) Less than or equal to 4minutes.

8 a) Use Trapezoidal rule to evaluate $\int_0^1 x^3\,dx$ considering five subintervals (4)

b) Using Newton's forward interpolation formula, find y at $x = 8$ from the following table: (8)

x :	0	5	10	15	20	25
y :	7	11	14	18	24	32

c) Using Euler's method, solve for y at $x = 0.1$ from $\frac{dy}{dx} = x + y + xy, \quad y(0) = 1$ taking step size $h = 0.025$. (8)

9 a) The transition probability matrix of a Markov chain $\{X_n, n \geq 0\}$ having three states 1, 2 and 3 is $P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.6 & 0.3 \\ 0.4 & 0.3 & 0.3 \end{bmatrix}$ and the initial probability distribution is $p(0) = [\,0.5 \quad 0.3 \quad 0.2\,]$. Find the following: (10)

i) $P\{X_2 = 2\}$ ii) $P\{X_3 = 3, X_2 = 2, X_1 = 1, X_0 = 3\}$.

b) Using Newton-Raphson method, compute the real root of $f(x) = x^3 - 2x - 5$ correct to 5 decimal places. (5)

c) Using Lagrange's interpolation formula, find the values of y when $x = 10$ from the following table : (5)

x :	5	6	9	11
y :	12	13	14	16

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Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FOURTH SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2018

Course Code: MA204
Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS (AE, EC)

Max. Marks: 100 Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A
Answer any two questions

1 a) If the random variable X takes the values 1,2,3 and 4 such that 2P(X=1)=3P(X=2)=P(X=3)=5P(X=4), find the probability distribution and cumulative distribution function of X 7

b) A complex electronic system is built with a certain number of backup components in itssubsystems. One subsystem has four identical components, each with a probability of 0.2 offailing in less than 1000 hours. The subsystem will operate if any two of the four componentsare operating. Assume that the components operate independently. Find the probability that 8

i)exactly two of the four components last longer than 1000 hours.

ii)the subsystem operates longer than 1000 hours.

2 a) A gardener sows 4 seeds in each of 100 plant pots. The number of pots in which 0,1,2,3 and 4 of seeds germinated is given in the following table. Fit a binomial distribution to the data 7

No. of seeds germinated	0	1	2	3	4
No. of pots	13	35	34	15	3

b) Find the mean and variance for the pdf 8

$$f(x) = \begin{cases} kx^2, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

3 a) If a random variable X has the exponential distribution with mean ½, calculate the probabilities that i) X will lie between 1 and 3 ii) X is greater than 0.5 iii) X is atmost 4 7

b) In a normal distribution, 10% of the items are below 55 and 20% are above 59. Find the mean and standard deviation of the distribution. What percentage of the items are above 60? 8

PART B

Answer any two questions

4 a) Obtain the distribution function of a continuous two dimensional random variable (X,Y) with the joint pdf given by 7

$$f(x,y) = \begin{cases} e^{-x-y}, & 0 < x < \infty \ , \ 0 < y < \infty \\ 0, & \text{elsewhere} \end{cases}$$

b) If the joint pdf of (X,Y) is given by 8

$$f(x,y) = \begin{cases} ½, & x > 0, y > 0, x + y < 2 \\ 0, & \text{otherwise} \end{cases}$$

Find $P\{X \leq 1, Y \leq 1\}$, $P\{ X + Y < 1 \}$ and $P\{ X > 2Y \}$

5 a) The joint pdf of (X,Y) is 7

$$f(x,y) = \begin{cases} 8xy, & 0 < y < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

i) Check whether X and Y are independent

ii) Find $P(X + Y < 1)$

b) Prove that the power spectral density and autocorrelation function of a real WSS process form a Fourier cosine transform pair 8

6 a) If $X(t) = P + Qt$ is the random process where P and Q are independent random variables with $E(P)= p$, $E(Q)= q$, $Var(P)= \sigma_1^2$, $Var (Q) = \sigma_2^2$ then find the mean, autocorrelation and autocovariance of the process 7

b) Consider the random process $X(t)= A \cos(\omega t) + B \sin (\omega t)$ where A and B are independent random variables with mean 0 and equal variance. Show that X(t) is a WSS. 8

PART C

Answer any two questions

7 a) Cell phone calls processed by a certain wireless base station arrive according to a Poisson process with an average of 12 per minute. i) What is the probability that more than two calls arrive in an interval of length 20 seconds ii) What is the probability that more than 2 calls arrive in each of two consecutive intervals of length 30 seconds 7

b) Show that the time between any two consecutive occurrences of a Poisson process is a random variable following an exponential distribution 7

c) Find the mean, variance, autocorrelation and autocovariance of a Poisson process 6

8 a) Find the positive solution of the equation $2\sin x = x$ using Newton-Raphson method 6

b) Using Newton's forward difference interpolation formula evaluate f(2.05) from the following table 6

X	2.0	2.1	2.2	2.3	2.4
f(x)	1.414214	1.449138	1.483240	1.516575	1.549193

c) Find an approximate value of $\log_e 5$ by evaluating $\int_0^5 \frac{dx}{4x+5}$ using Simpson's $1/3^{rd}$ rule with h=0.5 8

9 a) The transition probability matrix of a Markov chain $\{X_n\}$, n =1,2,3,….. having three states 1,2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 04 \\ 0.3 & 0.4 & 0.3 \\ 0.6 & 0.2 & 0.2 \end{bmatrix}$ 10

And the initial distribution is

P(0)= [0.7 0.2 0.1]. Find $P(X_2=3)$

b) Evaluate $\int_0^1 \frac{dx}{1+x}$ using trapezoidal rule 5

c) Using Runge-Kutta method of order four, find y(0.2) given that $\frac{dy}{dx}$ = y – x, y(0)=2 by taking h=0.1 5

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FOURTH SEMESTER B.TECH DEGREE EXAMINATION(R&S), MAY 2019

Course Code: MA204
Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS (AE, EC)

Max. Marks: 100 Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A

Answer any two full questions, each carries 15 marks

1 a) The probability mass function of a random variable X is given below: (7)

x	0	1	2	3
$f(x)$	c	$2c^2$	c^2	$3c^2$

Determine (i) the value of c (ii) $P(X \geq 1)$

(iii) $P[X > 1/(0 < X < 3)]$ (iv) $E(X)$

b) The probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20, find the number of packets containing (i) atleast 2 (ii) exactly 2 (ii) atmost 2 defective items in a consignment of 1000 packets using Poisson distribution. (8)

2 a) The mileage which a car owner gets with a certain kind of tyre is a random variable having an exponential distribution with mean 60,000 km Find the probabilities that one of these tyres will last, (7)
(a) at least 55,000 km (b) atmost 65,000 km

b) A random variable X $follows$ uniform distribution in $(-3,3)$. Find (8)

(i) $P(|X| < 2)$ (ii) $P(|X-2| < 2)$ (iii) $P(|X| > 1)$

(iv) the value of K for which $P(X > k) = \frac{1}{3}$

3 a) Fit a binomial distribution to the following data: (7)

x	0	1	2	3	4	5	6
f_i	5	18	28	12	7	6	4

b) In an examination 30% of the candidates obtained marks below 40 and 10% of the candidates got above 75 marks. Assuming that the marks are normally distributed, find the mean and standard deviation of the distribution. (8)

PART B

Answer any two full questions, each carries 15 marks

4 a) The life time of a certain type of electric bulbs may be considered to follow exponential distribution with mean 50 hrs. Use central limit theorem to find the approximate probability that 100 of these electric bulbs will provide a total of more than 6000 hrs of burning time. (7)

b) The joint density function of two continuous random variables X, Y is given by (8)

$$f(x,y) = \begin{cases} K(1-x-y), & 0 < x < \frac{1}{2}; 0 < y < \frac{1}{2} \\ 0, & otherwise \end{cases}.$$

Find (i) the value of K $(ii) P\left(X < \frac{1}{4}, Y > \frac{1}{4}\right)$ (iii) the marginal distributions of X, Y (iv) check whether X, Y are independent.

5 a) Let $X(t) = A\cos\omega t - B\sin\omega t$, where A and B are independent random variables following $N(0, \sigma^2)$. Then show that $\{X(t)\}$ is WSS. (7)

b) Find the power spectral density function of the WSS process whose autocorrelation function is $e^{-\alpha\tau^2}$. (8)

6 a) The joint probability distribution of X and Y is given by $f(x,y) = \frac{2x+3y}{54}$ for $x = 1,2;\ y = 1,2,3.$ Find (i) the marginal distributions of X and Y (ii) The conditional distribution of X for $Y = y$. (7)

b) The power spectral density of a WSS process is $\frac{\omega^2+9}{\omega^4+5\omega^2+4}$. Find the autocorrelation function and power of the process. (8)

PART C

Answer any two full questions, each carries 20 marks

7 a) The tpm of a Markov chain with 4 states 0, 1, 2, 3 is given by (10)

$$P = \begin{bmatrix} 0.2 & 0.8 & 0 & 0 \\ 0 & 0.2 & 0.8 & 0 \\ 0 & 0 & 0.2 & 0.8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

with initial distribution $P\{X_0 = i\} = \frac{1}{3}, i = 0,1,2,3.$

Find $(i) P\{X_1 = 2/\ X_0 = 1\}$ $(ii)\ P\{X_2 = 3/X_0 = 1\}$

$(iii)\ P(X_2 = 3, X_1 = 2, X_0 = 2)$ $(iv)\ P\{X_2 = 3\}$

b) The tpm of a Markov Chain is $P = \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$. Find the steady state distribution of the chain. (5)

c) A radioactive source emits particles at the rate of 6 per minutes in accordance with Poisson process. Each particle emitted has a probability of $\frac{1}{3}$ being recorded. Find the probability that atleast 5 particles are recorded in 5 minutes. (5)

8 a) Evaluate $\int_4^{5.2} log_e(x)\ dx$ using Simpson's 1/3rd rule . (Take $h = 0.2$) (7)

b) Use Newton's forward interpolation formula to evaluate $y(23)$ from the following data: (7)

x	20	25	30	35	40	45
y	34.3	32.1	29.3	25.6	22.7	21.9

c) Use Runge-Kutta method of order 4 to find $y(0.2)$ for the differential equation: (6)

$y' = 3x + 0.5y,\ y(0) = 1$. (Take h = 0.2)

9 a) A house wife buys 3 types of cereals A, B, and C. She never buys the same cereals in successive weeks. If she buys cereal A, next week she buys B. However, if she buys B or C, next week she is 3 times as likely to buy A as the other cereals. In the first week of May she buys cereal C. Then what is the probability that (i) in the second week she buys cereal A (ii) In the third week she buys cereal C (iii) In the long run, how often she buys cereal B. (10)

b) Use Lagrange's interpolation formula to find $y(2)$ from the following table: (5)

x	1	3	4
y	1	27	64

c) Evaluate cube root of 41 correct to four decimal places using Newton- Raphson method correct to 4 decimal places. (5)

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Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FOURTH SEMESTER B.TECH DEGREE EXAMINATION(S), DECEMBER 2019

Course Code: MA204

Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS

Max. Marks: 100 Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A

Answer any two full questions, each carries 15 marks

1 a) A coin is biased so that the head is twice as likely to appear as the tail. The coin is tossed twice. Find the expected value of the number of heads. Also find the variance of number of heads. (7)

b) Show that Poisson distribution is the limiting case of Binomial distribution as $n \longrightarrow \infty, p \longrightarrow 0$. (8)

2 a) The time required to repair a machine is exponentially distributed with parameter $\frac{1}{2}$. What is the probability that (i) repair time exceeds 2 hrs (ii) repair time is between 3 hrs and 5 hrs? (7)

b) A random variable X has the following probability density function: $f(x) = kx(2-x), 0 < x < 2$. Find (i) the value of k (ii) mean (iii) variance (iv) distribution function. (8)

3 a) Fit a Poisson distribution to the following data: (7)

x	0	1	2	3	4	5
f_i	142	156	69	27	5	1

b) In an intelligence test administrated on 1000 children, the average was 60 and the standard deviation was 20. Assuming that the marks obtained by the children follow a normal distribution, find the number of children who have scored (i) above 90 marks (ii) below 40 marks (iii) between 50 and 80 marks (8)

PART B

Answer any two full questions, each carries 15 marks

4 a) Let $X_1, X_2, \ldots\ldots, X_{75}$ are independently and identically distributed random variables following Poisson distribution with parameter $\lambda = 2$. Use Central Limit Theorem to estimate $P(120 \leq S_{75} \leq 160)$, where $S_{75} = X_1 + X_2 + \ldots\ldots + X_{75}$. (7)

b) The joint density function of two continuous random variables X, Y is given by (8)

$$f(x,y) = \begin{cases} Ke^{-2x-2y}, & x \geq 0, y \geq 0 \\ 0, & otherwise \end{cases}.$$

Find (i) the value of K $(ii) P(X > 1)$ (iii) the marginal distributions of X, Y and (iv) check whether X, Y are independent.

5 a) Find the power spectral density function of the WSS process whose autocorrelation function is $A^2 e^{-2\alpha|\tau|}$. (7)

b) Let $X(t) = A\cos(50t + \theta)$, where A and θ are independent random variables. A is a random variable with mean 0 and variance 1 and θ is uniformly distributed in $(-\pi, \pi)$. Show that $\{X(t)\}$ is WSS. (8)

6 a) If $\{X(t)\}$ is a random process with mean 3 and $R(t_1, t_2) = 9 + 4e^{-|t_1 - t_2|/5}$. Find $(i) V[X(5)]$ $(ii) V[X(8)]$ $(iii) Cov[x(5), X(8)]$ (7)

b) 3 balls drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. Let X denotes number of white balls drawn and Y denotes number of red balls drawn. Find the joint pdf of X and Y, find the marginal pdfs and check whether X and Y are independent. (8)

PART C

Answer any two full questions, each carries 20 marks

7 a) The tpm of a Markov chain with 3 states 1, 2, 3 is given by $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ with initial distribution $P\{0\} = [0.7, 0.2, 0.1]$. (10)

Find $(i) P\{X_1 = 2 / X_0 = 1\}$ $(ii) P(X_3 = 3, X_2 = 2, X_1 = 1 \ X_0 = 2)$

$(iii) P\{X_2 = 3 / X_0 = 1\}$ $(iv) P\{X_2 = 3\}$

b) The tpm of a Markov Chain is $P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$. Find the steady state distribution of the chain. (5)

c) Suppose that customers arrive at a bank according to a Poisson process with mean rate of 3 per minute. Find the probability that during a time interval of 2 minutes (5)

(i) exactly 4 customers arrive (ii) more than 4 customers arrive.

8 a) Use Newton's forward interpolation formula to find the interpolating polynomial for the following data. Hence evaluate $y(1.5)$. (7)

x	0	1	2	3
y	0	2	6	18

b) Solve using Runge - Kutta method of order 4: (7)

$y' = 8.5 - 20x + 12x^2 - 2x^3, y(0) = 1$ *for* $x = 0.5$. [Choose $h = 0.5$]

c) Evaluate $\int_0^2 xe^x dx$ using Simpson's 1/3rd rule with $n = 8$. (6)

9 a) A gambler has rupees 2 and he plays a betting game where he wins Rs. 1, if a tail shows up and loses Rs. 1 if a head shows up in the tossing of a fair coin. He stops playing this game if he wins Rs 2 or loses Rs 2. Find (i) the transition probability matrix of the Markov chain (ii) the probability that the gambler lost his money at the end of 2 plays.(iii) the probability that the gambler ends the game in 2 plays. (10)

b) Using Lagrange's interpolation method to find the value of $f(2)$ from the following data: (5)

x	0	1	3
y	1	3	55

c) Find a positive root of the equation $x^3 + x - 1 = 0$ using Newton-Raphson method correct to 4 decimal places. (5)

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth semester B.Tech examinations (S), September 2020

Course Code: MA204

Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS (AE, EC)

Max. Marks: 100 Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A

Answer any two questions

1 a) Let X be a random variable taking values $-1, 0$ and 1 such that $P(X=-1) = 2P(X=0) = P(X=1)$. Find (i) the pdf of X (ii) the mean of $(2X-5)$. 7

b) A pair of dice is thrown 5 times. If getting a doublet is considered to be a success, use Binomial distribution to find the probability of getting (i) atleast 2 successes (ii) at most 2 successes (iii) exactly 2 failures. 8

2 a) If $f(x) = kx^2e^{-x}, x > 0$, find k, mean and variance of the random variable. 7

b) The length of time (in minutes) a person speaks over the phone follows an exponential distribution with mean 4. Find the probability that the person will talk for (i) more than 8 minutes (ii) between 3 and 6 minutes (iii) less than 2 minutes. 8

3 a) Fit a Poisson distribution to the following data: 7

x	0	1	2	3	4	5	6
f	167	70	35	17	7	3	1

b) The marks obtained by students in an intelligence test follow normal distribution with mean 45 and standard deviation 25. Find the percentage of students who scored marks (i) more than 80 (ii) between 30 and 70 (iii) below 35 8

PART B

Answer any two questions

4 a) A random sample of size 200 is taken from a population whose mean is 50 and variance is 600. Using Central Limit theorem, find the probability that the mean of the sample $\bar{X}$ will not differ from $\mu = 50$ by more than 5. 7

b) The joint pdf of X, Y is given by $f(x,y) = k(x+2y), x = 1,2,3; y = 1,2,3$. Find (i) k (ii) marginal pdf of X, Y (iii) $P(X < 3, Y \geq 2)$. 8

5 a) The power spectral density function of a random telegraph signal process is $S(\omega) = \frac{8}{9+\omega^2}$. Find the corresponding auto correlation function and the power of the process. 7

b) $\{X(t)\}$ is a random process with mean 2 and auto correlation 8

$R(t_1, t_2) = 5 + 3e^{-0.1|t_1 - t_2|}$. Find the mean, variance and the covariance of the random variables $X(4)$ and $X(6)$.

6 a) The joint PDF of X, Y is $f(x, y) = kxye^{-(x^2+y^2)}, x > 0; y > 0$. Find the value of k, marginal distributions of X, Y and check whether X, Y are independent. 7

b) Let $\{X(t) = A\cos\omega t + Bsin\omega t\,, t > 0\}$ be a random process where A and B are independent random variables following normal distribution with mean 0 and variance 4. Check whether $\{X(t)\}$ is WSS. 8

PART C

Answer any two questions

7 a) Is the Poisson Process a stationary process? Find the Autocorrelation of the Poisson process. 7

(b) The tpm of a Markov chain with states 1,2,3 is $P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.6 & 0.3 \\ 0.4 & 0.3 & 0.3 \end{bmatrix}$ and the initial distribution is P(0)= $(0.5, 0.3, 0.2)$. Find $(i) P(X_2) = 2$ (ii) $P(X_3 = 3, X_2 = 2, X_1 = 1, X_0 = 3)$ 8

c) The tpm of a Markov chain is $P = \begin{bmatrix} 0.5 & 0.5 \\ 0.1 & 0.9 \end{bmatrix}$. Find the steady state distribution of the process. 5

8 a) Use Newton's forward difference formula to find y at $x = 1.5$. 7

x	0	1	2	3	4
y	7	10	13	22	43

b) Use Euler's method with $h = 0.025$ to compute the value of $y(0.1)$ for the differential equation $y' = x - y^2,\ y(0) = 1$. 8

c) Use Newton-Raphson method to find $\sqrt{35}$ correct to 4 decimal places. 5

9 a) A man either drives a car or catches a train to go to office every day. He never goes two days in a row by train. But if he drives one day then the next day he is just as likely to drive again as he is to travel by train. On the first day of a week, the man tosses a fair dice and drives to work if he gets a 6. Find the Probability that (i) he takes train on second day (ii) he drives to work on third day (iii) he drives to work in the long run. 10

b) Find the Lagrange's Interpolating polynomial corresponding to given data: $f(0) = 0, f(1) = 1, f(2) = 20$. Hence find $f(1.5)$ 5

c) Evaluate $\int_0^1 e^{-x^2}\,dx$ using Trapezoidal rule with $h = 0.25$ 5