

Reg. No. _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FOURTH SEMESTER B.TECH DEGREE EXAMINATION, MAY 2017

MA202: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND NUMERICAL METHODS

Max. Marks: 100

Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A (MODULES I AND II)

Answer two full questions.

1. a. Given that $f(x) = \frac{k}{2^x}$ is a probability distribution of a random variable that can take on the values $x = 0, 1, 2, 3$ and 4 , find k . Find the cumulative distribution function. (7)
 b. If 6 of the 18 new buildings in a city violate the building code, what is the probability that a building inspector who randomly select 4 of the new buildings will catch
 - i) none of the new buildings that violate the building code
 - ii) one of the new buildings that violate the building code
 - iii) at least two of the new buildings violate the building code (8)
2. a. Prove that binomial distribution with parameters n and p can be approximated to Poisson distribution when n is large and p is small with $np = \lambda$ a constant. (7)
 b. Find the value of k for the probability density $f(x)$ given below and hence find its mean and variance where

$$f(x) = \begin{cases} kx^3 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \quad (8)$$
3. a. A random variable has normal distribution with $\mu = 62.4$. Find its standard deviation if the probability is 0.2 that it will take on a value greater than 79.2 (7)
 b. The amount of time that a surveillance camera will run without having to be reset is a random variable having the exponential distribution with the parameter 50 days. Find the probability that such a camera will
 - i) have to be reset in less than 20 days
 - ii) not have to be reset in at least 60 days. (8)

PART B (MODULES III AND IV)

Answer two full questions.

4. a. Use Fourier integral to show that $\int_0^{\infty} \frac{\cos x\omega + \omega \sin x\omega}{1 + \omega^2} d\omega = \begin{cases} 0 & \text{if } x < 0 \\ \pi/2 & \text{if } x = 0 \\ \pi e^{-x} & \text{if } x > 0 \end{cases} \quad (7)$

- b. Represent $f(x) = \begin{cases} x^2 & 0 < x < 1 \\ 0 & x > 1 \end{cases}$ as a Fourier cosine integral. (8)

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FOURTH SEMESTER B.TECH DEGREE EXAMINATION, JULY 2017

Course Code: MA202

Course Name: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND NUMERICAL METHODS

Max. Marks: 100

Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A (MODULES I AND II)

Answer two full questions.

- 1 a) A random variable X has the following probability mass function (8)
- | | | | | | | | | |
|-------|---|---|----|----|----|----------------|-----------------|---------------------|
| X: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| P(x): | 0 | k | 2k | 2k | 3k | k ² | 2k ² | 7k ² + k |
- Find (i) value of k (ii) P(0 < x < 5) (iii) P(x ≥ 6)
- b) An insurance company agent accepts policies of 5 men, all of identical age and good health. Probability that a man of this age will be alive 30 years is $\frac{2}{3}$. Find the probability that in 30 years (i) all 5 men (ii) at least one men will be alive. (7)
- 2 a) Show that for a poisson distribution with parameter λ , mean = variance = λ (7)
- b) In a given city 6% of all drivers get at least one parking ticket per year. Use the poisson approximation to the binomial distribution to determine the probabilities that among 80 drivers (randomly chosen in this city) (8)
- (i) 4 will get at least one parking ticket in any given year
(ii) at least 3 will get at least one parking ticket in any given year
(iii) anywhere from 3 to 6 inclusive, will get at least one parking ticket in any given year.
- 3 a) The marks obtained in mathematics by 1000 students are normally distributed with mean 78% and standard deviation 11%. Determine (8)
- (i) How many students got marks above 90%
(ii) What was the highest mark obtained by the lowest 10% of students
- b) Derive the mean and variance of the uniform distribution in the interval (a,b) (7)

PART B (MODULES III AND IV)

Answer two full questions.

- 4 a) Express $f(x) = 1, 0 < x < \pi$ (7)
- $0, x > \pi,$
- a Fourier sine integral and evaluate $\int_0^{\infty} \frac{1 - \cos \pi \omega}{\omega} \sin x \omega \, d\omega$
- b) Using Fourier integral representation show that (8)
- $$\int_0^{\infty} \frac{\sin \omega - \omega \cos \omega}{\omega^2} \sin x \omega \, d\omega = \begin{cases} \frac{\pi}{2} x, & \text{if } 0 < x < 1 \\ \frac{\pi}{4}, & \text{if } x = 1 \\ 0, & \text{if } x > 1 \end{cases}$$

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- 5 a) Find the Fourier cosine transform of (7)
 $f(x) = x^2$, if $0 < x < 1$
 0 , if $x > 1$
- b) Find the Laplace transform of (8)
 (i) $\sinh t \cos t$ (ii) $(t-1)^3$
- 6 a) Find the inverse Laplace transform of $\frac{1}{(s + \sqrt{2})(s - \sqrt{3})}$ (7)
- b) Solve the initial value problem, using Laplace transforms. (8)
 $y'' + y' + 9y = 0$, $y(0) = 0.16$, $y'(0) = 0$

PART C (MODULES V AND VI)*Answer two full questions.*

- 7 a) Using Newton Raphson Method Compute the square root of 51 correct to 4 decimal (7)
 places
- b) For the following data calculate the value of y when x = 9 (7)
 $x : 8 \quad 10 \quad 12 \quad 14 \quad 16 \quad 18$
 $y : 10 \quad 19 \quad 32.5 \quad 54 \quad 89.5 \quad 154$
- c) Given $f(2) = 5$, $f(2.5) = 6$, find the linear interpolating polynomial using Lagrange's (6)
 formula and also find $f(2.2)$
- 8 a) Determine the interpolating polynomial for the following data (6)
 $x : -1 \quad 0 \quad 1 \quad 3$
 $y : 2 \quad 1 \quad 0 \quad -1$ Hence find the value of y when x = 2
- b) Solve the following by Gauss – Seidel Method (8)
 $6x + 15y + 2z = 72$
 $x + y + 54z = 110$
 $27x + 6y - z = 85$
- c) Evaluate $\int_0^6 \frac{dx}{1+x^2}$, using Simpsons rule by taking step size h=1 (6)
- 9 a) Using Euler Method, Solve $y' = x + y$, $y(0) = 1$ for $x = 0.2$ (6)
- b) Find $y(0.1)$ by improved Euler method given $y = -xy^2$, $y(0) = 2$ (6)
- c) Apply Runge – Kutta fourth order method to find an approximate value of y when (8)
 $x = 0.1$ given that $\frac{dy}{dx} = x + y$ and $y = 1$
 when $x = 0$

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FOURTH SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2018

Course Code: MA202

Course Name: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND NUMERICAL METHODS

Max. Marks: 100

Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A (MODULES I AND II)

Answer two full questions.

- 1 a) Suppose that the probabilities are 0.4, 0.3, 0.2, and 0.1 that there will be 0, 1, 2, or 3 power failures in a certain city during the month of July. Find the mean and variance of this probability distribution. (7)
- b) During one stage in the manufacture of integrated circuit chips, a coating must be applied. If 70% of chips receive a thick enough coating. Use Binomial distribution to find the probabilities that, among 15 chips
- (i) at least 12 will have thick enough coating;
 - (ii) at most 6 will have thick enough coating;
 - (iii) exactly 10 will have thick enough coating.
- 2 a) If the distribution function of a random variable is given by (7)
- $$F(x) = \begin{cases} 1 - \frac{1}{x^2} & \text{for } x > 1 \\ 0 & \text{for } x \leq 1 \end{cases}$$
- find the probabilities that this random variable will take on a value
- (i) less than 3;
 - (ii) between 4 and 5.
- b) In a given city, 6% of all drivers get at least one parking ticket per year. Use the Poisson approximation to the binomial distribution to determine the probabilities that among 80 drivers (randomly chosen in the city):
- (i) 4 will get at least one parking ticket in any given year;
 - (ii) at least 3 will get at least one parking ticket in any given year;
 - (iii) anywhere from 3 to 6, inclusive, will get at least one parking ticket in any given year.
- 3 a) Derive mean and variance of uniform distribution. (7)
- b) The time required to assemble a piece of machinery is a random variable having approximately a normal distribution with mean 12.9 minutes and standard deviation 2.0 minutes. What are the probabilities that the assembly of a piece of machinery of this kind will take
- (i) at least 11.5 minutes;
 - (ii) anywhere from 11.0 to 14.8 minutes?

PART B (MODULES III AND IV)

Answer two full questions.

- 4 a) Using Fourier cosine integral, show that $\int_0^{\infty} \frac{\cos xw}{1+w^2} dw = \frac{\pi}{2} e^{-x}$ if $x > 0$. (7)
- b) Find the Fourier sine transform of $f(x) = \begin{cases} \sin x & \text{if } 0 < x < \pi \\ 0 & \text{if } x > \pi \end{cases}$. (8)

- 5 a) Find the Fourier transform of $f(x) = \begin{cases} e^{kx} & \text{if } x < 0 \\ 0 & \text{if } x > 0 \end{cases}$, $k > 0$. (7)
- b) Find the inverse Laplace transform of $\frac{5}{(s^2 + 1)(s^2 + 25)}$ using Convolution Theorem. (8)
- 6 a) Find the Laplace transforms of (i) $t e^{kt}$ (ii) $\cos(\omega t + \theta)$ (7)
- b) Solve the initial value problem $y'' - y' - 6y = 0$, $y(0) = 6$, $y'(0) = 13$ by using Laplace transforms. (8)

PART C (MODULES V AND VI)

Answer two full questions.

- 7 a) Find the positive solution of $2 \sin x = x$ by using Newton-Raphson method, the solution is near to 2. (7)
- b) Calculate the Lagrange polynomial $p(x)$ for the 4-D values of the function $f(x)$, $f(1.00) = 1.0000$, $f(1.02) = 0.9888$, $f(1.04) = 0.9784$, and from it find the approximate value of $f(x)$ at $x = 1.005$. (7)
- c) Compute $f(1.5)$ from $f(1) = -1$, $f(2) = -1$, $f(3) = 1$, $f(4) = 5$ by using Newton's forward interpolation formula. (6)
- 8 a) Solve $6x_1 + 2x_2 + 8x_3 = 26$, $3x_1 + 5x_2 + 2x_3 = 8$, $8x_2 + 2x_3 = -7$ by Gauss Elimination method. (7)
- b) Find the value of $(13)^{1/3}$ using Newton Raphson method. (7)
- c) Evaluate $\int_0^1 e^{-x^2} dx$ by Trapezoidal rule taking 10 subintervals. (6)
- 9 a) Use Euler's method with $h = 0.1$, compute the value of $y(0.5)$ for the equation $y' = (y + x)^2$, $y(0) = 0$. (7)
- b) Use Runge-Kutta method with $h = 0.1$, compute the value of $y(0.1)$ for the equation $y' = xy^2$, $y(0) = 1$. (7)
- c) Evaluate $\int_0^1 \frac{dx}{\cos^2 x}$ by Simpson's rule taking 10 subintervals and compare it with the exact solution. (6)

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FOURTH SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

Course Code: MA202

Course Name: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND NUMERICAL METHODS

Max. Marks: 100

Duration: 3 Hours

(Normal distribution table is allowed in the examination hall)

PART A (MODULES I AND II)

Answer any two full questions, each carries 15 marks

Marks

- 1 a) Derive the formula for mean and variance of Binomial distribution. (7)
- b) 100 fair dice are thrown. Find the expectation of the sum of the numbers thrown. (8)
- 2 a) A continuous random variable X has a pdf $f(x) = kx^2e^{-x}; x \geq 0$. (7)
 Find i) Value of k and ii) Mean of the distribution.
- b) If X is a uniformly distributed R V with mean 1 and variance $\frac{4}{3}$, find $P(|X - 2| < 2)$ (8)
- 3 a) The time in hours required to repair a machine is exponentially distributed with mean 20. What is the Probability that the required time : (7)
 i) Exceeds 30 hrs ii) Between 16 hrs and 24 hrs.
- b) Marks of a set of students for a certain subject are approximately normally distributed with mean 62 and variance 9. If 4 students are randomly selected, what is the probability that 3 of them have less than 60 marks? (8)

PART B (MODULES III AND IV)

Answer any two full questions, each carries 15 marks

- 4 a) Find the Fourier Integral representation of $f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$ (7)
- b) Find the Fourier Sine Transform of $f(x) = e^{-|x|}$. Hence evaluate $\int_0^{\infty} \frac{\omega \sin \omega x}{1 + \omega^2} d\omega$. (8)
- 5 a) Find the Laplace Transform of : (7)
 (i) $\sin 3t \cos 2t$ (ii) $e^{-2t} \cos^2 t$
- b) Find the Inverse Laplace Transform of: (8)
 (i) $\frac{s-4}{s^2-4}$ (ii) $\frac{4}{s^2-2s-3}$
- 6 a) Find the Fourier Cosine Transform of $f(x) = \sin x; 0 < x < \pi$. (7)
- b) Solve, by using Laplace Transform: $y'' + y = 3 \cos 2t; y(0) = 0, y'(0) = 0$. (8)

PART C (MODULES V AND VI)

Answer any two full questions, each carries 20 marks

- 7 a) Find a root lying between 0 and $\frac{\pi}{2}$ of $f(x) = \cos x - 3x + 1 = 0$. (correct to 3 decimal places). (6)
- b) Using Lagrange's interpolation formula, fit a polynomial to the given data and hence find $y(2)$ (7)

x	1	3	4
y	1	27	64

- c) Using Newton's Forward Interpolation Formula, find the value of $\sin 52^\circ$ given that $\sin 45^\circ = 0.7071$, $\sin 50^\circ = 0.7660$, $\sin 55^\circ = 0.8192$, $\sin 60^\circ = 0.8660$, $\sin 65^\circ = 0.9063$. (6)
- 8 a) Solve the following equations by Gauss- Seidel iteration Method. (correct to 3 decimal places). (7)

$$27x + 6y - z = 85, \quad 6x + 15y + 2z = 72, \quad x + y + 54z = 110.$$

- b) Use Euler's Method with $h = 0.025$, compute the value of $y(0.1)$ for $y' = x - y^2$; $y(0) = 1$. (7)

- c) A river is 80m wide. The depth y in meters at a distance x meter from one bank is given by the following table. (6)

x	0	10	20	30	40	50	60	70	80
y	0	5	8	10	15	12	7	3	1

Find approximately the area of cross section using Simpson's $1/3$ rd rule.

- 9 a) Using Newton-Raphson Method, derive a formula to find $\sqrt[3]{N}$ where N is a real number. Hence evaluate $\sqrt[3]{35}$ correct to three decimal places. (10)
- b) Using Runge- Kutta Method of Fourth Order, $\frac{dy}{dx} = \sqrt{x + y}$; $y(0) = 1$, find $y(0.2)$ with $h = 0.1$ (10)

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**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FOURTH SEMESTER B.TECH DEGREE EXAMINATION(R&S), MAY 2019**

Course Code: MA202

Course Name: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND NUMERICAL METHODS

Max. Marks: 100

Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A (MODULES I AND II)

Answer two full questions.

- 1 a) A random variable X takes the values -3,-2,-1,0,1,2,3 such that $P(X=0)=P(X>0) = P(X<0)$ and $P(X=-3) = P(X=-2) = P(X=-1) = P(X=1) = P(X=2) = P(X=3)$. Obtain the probability distribution and the distribution function of X (7)
- b) If the sum of the mean and variance of a binomial distribution for 5 trials is 1.8 Find the probability distribution function. (8)
- 2 a) It is known that 2% of the accounts in a company are delinquent. If 5 accounts are selected at random, compute the following probabilities (i) atmost 2 accounts will be delinquent (ii) atmost 4 accounts will be delinquent (7)
- b) Find the value of k and hence find the mean and variance of the distribution (8)

$$f(x) = kx^2e^{-x} \quad 0 < x < \infty$$
- 3 a) If X is uniformly distributed over $(-\alpha, \alpha)$, $\alpha < 0$. Find α so that (i) $P(x > 1) = 1/3$ (7)

(ii) $P(|x| < 1) = P(|x| > 1)$
- b) 5% of the observation in a normal distribution are below 5 and 25% of the observations are between 5 and 25. Find mean and SD (8)

PART B (MODULES III AND IV)

Answer two full questions.

- 4 a) Find the fourier transform of $f(x) = \begin{cases} 1 - |x| & \text{if } |x| \leq 1 \\ 0 & \text{if } |x| > 1 \end{cases}$ and also find fourier inverse transform (7)
- b) Using fourier sine integral for $f(x) = e^{-ax}$ show that $\int_0^{\infty} \frac{\lambda \sin \lambda x}{\lambda^2 + a^2} d\lambda = \pi e^{-ax}$ (8)
- 5 a) Find the fourier sine transform of e^{-x} , $x \geq 0$. Hence evaluate $\int_0^{\infty} \frac{x \sin x}{1+x^2} dx$ (7)

- b) Find the Laplace transform of (i) $te^{-t}\sin t$ (ii) $\frac{\sin^2 t}{t}$ (8)
- 6 a) Solve $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 5y = 4e^{3t}$ given that $y = 2, \frac{dy}{dt} = 7$ when $t = 0$ (7)
- b) Using convolution theorem find $L^{-1} \frac{s}{(s^2+a^2)^2}$ (8)

PART C (MODULES V AND VI)

Answer two full questions.

- 7 a) Using Newton Raphson method find correct to four decimal places, the root (8)
between 0 and 1 of the equation $x^3 - 6x + 4 = 0$
- b) The population of a town is as follows (12)
- | Year | 1941 | 1951 | 1961 | 1971 | 1981 | 1991 |
|--------------------------|------|------|------|------|------|------|
| Population
(in lakhs) | 20 | 24 | 29 | 36 | 46 | 51 |
- Estimate the population increase during the period 1946 to 1976
- 8 a) Apply Lagrange's formula to obtain the value of y when $x=35$ given that (6)
- | | | | | |
|---|-----|-----|----|----|
| x | 30 | 34 | 38 | 42 |
| y | -30 | -13 | 3 | 18 |
- b) Solve the equation using Gauss elimination method (7)
 $2x + y + z = 10, 3x + 2y + 3z = 18, x + 4y + 9z = 16$
- c) Solve the system of equations $4x + 2y + z = 14, x + 5y - z = 10, x + y + 8z = 20$ (7)
using Gauss-Seidal iteration method
- 9 a) A solid of revolution is formed by rotating about the x axis, the area between the x (7)
axis, the line $x=0$ and $x=1$ and a curve through the points with the following
coordinates
- | | | | | | |
|---|--------|-------|-------|-------|-------|
| X | 0.0 | 0.25 | 0.50 | 0.75 | 1.00 |
| Y | 1.0000 | .9896 | .9589 | .9089 | .8415 |
- Estimate the volume of the solid formed using Trapezoidal rule
- b) Using Euler's method find $y(0.2)$ and $y(0.4)$ given $\frac{dy}{dx} = x + y, y(0) = 1$ and $h = 0.2$ (6)
- c) Use the fourth order Runge-Kutta method to find $y(0.2)$ from $\frac{dy}{dx} = y - x, y(0) = 2$ (7)
taking $h=0.1$

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FOURTH SEMESTER B.TECH DEGREE EXAMINATION(S), DECEMBER 2019

Course Code: MA202

Course Name: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND NUMERICAL METHODS

Max. Marks: 100

Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A (MODULES I AND II)

Answer two full questions.

- 1 a) The following table gives the probability that a certain computer will malfunction 0, 1, 2, 3, 4, 5, or 6 times on any one day

Number of Malfunctions	x	0	1	2	3	4	5	6	
Probability	f(x)	0.17	0.29	0.27	0.16	0.07	0.03	0.01	(7)

Find (i) The Mean, Variance and Standard Deviation of this probability distribution

- (ii) $P(0 < x < 5)$ (iii) $P(x > 4)$

- b) It is known that 5% of the books bound at a certain bindery have defective bindings. Find the probability that atmost 2 of 100 book bound by this bindery will have defective binding using

- (i) The formula for binomial distribution
(ii) Poisson approximation to the binomial distribution

- 2 a) Derive the mean, variance and distribution function of the uniform distribution in the interval (a,b). (7)

- b) The amount of time that a surveillance camera will run without having to be reset is a random variable having the exponential distribution with mean 50 days.

Find the probability that such a camera will

- (i) have to be reset in less than 20 days (8)
(ii) not have to be reset in at least 60 days
(iii) have to be reset between 20 and 60 days.

- 3 a) The time required to microwave a bag of popcorn using the automatic setting can be treated as a random variable having a normal distribution with standard deviation 10 seconds. If the probability is 0.8212 that the bag will take less than 282.5 seconds to pop, find the probability that it will take longer than 258.3 seconds to pop. (7)

- b) Prove that binomial distribution with parameters n and p can be approximated to Poisson distribution when n is large and p is small with $np = \lambda$, a constant. (8)

PART B (MODULES III AND IV)

Answer two full questions.

4 a) Use Fourier integral to show that
$$\int_0^{\infty} \frac{\cos x\omega + \omega \sin x\omega}{1 + \omega^2} d\omega = \begin{cases} 0 & \text{if } x < 0 \\ \pi/2 & \text{if } x = 0 \\ \pi e^{-x} & \text{if } x > 0 \end{cases} \quad (7)$$

b) Find the Fourier Sine and Cosine Transform of $f(x) = \begin{cases} x^2 & \text{if } 0 < x < 1 \\ 0 & \text{if } x > 1 \end{cases} \quad (8)$

- 5 a) Find the Laplace Transform of :

(i) $e^{-t} \sin 3t \cos 2t$

(ii) $t^2 \cos \omega t \quad (7)$

(iii) $t^2 u(t-1)$

- b) Find the inverse Laplace Transform of :

(i) $\frac{1-7s}{(s-3)(s-1)(s+2)}$

(ii) $\ln \frac{s-a}{s-b} \quad (8)$

(iii) $\frac{e^{-3s}}{(s-1)^3}$

6 a) Find the Fourier Sine Transform of $f(x) = e^{-|x|}$. Hence evaluate $\int_0^{\infty} \frac{\omega \sin x\omega}{1 + \omega^2} d\omega. \quad (7)$

b) Solve by using Laplace Transform: $y'' + 2y' - 3y = 6e^{-2t}, y(0) = 2, y'(0) = -14 \quad (8)$

PART C (MODULES V AND VI)

Answer two full questions.

- 7 a) Find the positive solution of $2\sin x = x$ using Newton Raphson (method correct to five decimal places). (6)

- b) Find the value of $\tan 33^\circ$ by using Lagrange's formula for interpolation (7)

X	30°	32°	35°	38°
$\tan x$	0.5774	0.6249	0.7002	0.7813

- c) A second degree polynomial passes through the points (1,-1) (2,-1) (3, 1) (4, 5). Find the polynomial $f(x)$, Also find $f(1.2)$. (7)

- 8 a) A river is 80 metre wide. The depth y in metres at a distance x metres from one

bank is given by the following table. Find approximately the area of cross section.

X	0	10	20	30	40	50	60	70	80
Y	0	5	8	10	15	12	7	3	1

(6)

- b) Using Improved Euler method find y at $x = 0.1$ and $x = 0.2$ for the equation (7)

$$y' = y - \frac{2x}{y}, y(0) = 1.$$

- c) Solve the initial value problem $y' + y \tan x = \sin 2x, y(0) = 1$ at $x = 0.2$ using Runge- Kutta method. (7)

- 9 a) Solve the following system of equations using Gauss elimination method.

$$10x + y + z = 6$$

$$x + 10y + z = 6$$

$$x + y + 10z = 6$$

(6)

- b) Solve the system of equations using Gauss Seidel iteration method starting with the initial approximation $x = y = z = 1$. (7)

$$4x + 5z = 12.5$$

$$x + 6y + 2z = 18.5$$

$$8x + 2y + z = -11.5$$

- c) The population of a town is as follows

Year (x)	1941	1951	1961	1971	1981	1991
Population in lakhs(y)	20	24	29	36	46	51

(7)

Find the population increase during the period from 1946 to 1976

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth semester B.Tech examinations (S), September 2020

Course Code: MA202**Course Name: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND NUMERICAL METHODS**

Max. Marks: 100

Duration: 3 Hours

*Normal distribution table is allowed in the examination hall.***PART A (MODULES I AND II)***Answer two full questions.*

- 1 a) Let X be a discrete random variable with mean 10 and variance 25. Find the positive values of α and β such that $Y = \alpha X - \beta$ has mean 0 and variance 1. 7
- b) Derive the mean and variance of a Poisson Distribution. 8
- 2 a) If a continuous random variable has the probability distribution function 7
- $$f(x) = \begin{cases} ke^{-3x} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$
- then find (i) value of k (ii) $P[0 \leq X \leq 2]$ (iii) $P[X > 1.5]$
- b) In a Normal Distribution, if 6% of the items are below 60 and 39% are above 70, then find the mean and standard deviation. 8
- 3 a) Out of 2000 families with 4 children each, how many would you expect to have (i) at least one boy (ii) at most one boy 7
- b) If X follows a uniform distribution in $(-2, 2)$, then (i) find $P[|X - 1| \leq 2]$ (ii) find k for which $P[X > k] = \frac{1}{3}$ (iii) Distribution function 8

PART B (MODULES III AND IV)*Answer two full questions.*

- 4 a) Find the Fourier Sine Integral of $f(x) = \begin{cases} \sin x & \text{if } 0 \leq x \leq \pi \\ 0 & \text{if } x > \pi \end{cases}$ 7
- b) Find the Fourier Cosine Transform of $f(x) = e^{-4x}$. Hence deduce that 8
- $$\int_0^{\infty} \frac{\cos 2x}{x^2 + 16} dx = \frac{\pi}{8} e^{-8}$$
- 5 a) Using Convolution theorem, evaluate the Inverse Laplace Transform of $\frac{s}{(s^2 + 4)^2}$ 7
- b) Evaluate (i) $L[t \sin^2 2t]$ (ii) $L^{-1} \left[\frac{s+5}{s^2 + 4s + 13} \right]$ 8

- 6 a) Find the Fourier Transform of $f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$ Hence show that $\int_0^\infty \frac{\sin \omega}{\omega} d\omega = \frac{\pi}{2}$ 7

- b) Solve using Laplace Transform: $y'' - 3y' + 2y = 4$ given $y(0) = 2, y'(0) = 3$ 8

PART C (MODULES V AND VI)

Answer two full questions.

- 7 a) Using Lagrange's interpolation formula, find a parabola of the form $y = ax^2 + bx + c$ passing through the points (0,0), (2,4) and (3,12) 6

- b) Using Newton-Raphson Method, find the real root lying between 0 and 1 of $3x - \cos x - 1 = 0$. (Correct to three decimal places) 7

- c) Apply Lagrange's interpolation formula to find y at $x = 2$ for the following values for $y = f(x)$. Given $f(0) = -12, f(1) = 0, f(3) = 6$ and $f(4) = 12$. 7

- 8 a) Solve by Gauss Elimination Method: 6

$$3x + 4y + 5z = 18, \quad 2x - y + 8z = 13, \quad 5x - 2y + 7z = 20.$$

- b) Evaluate $I = \int_0^6 \frac{1}{1+x} dx$ using (i) Trapezoidal Rule (ii) Simpson's $\frac{1}{3}$ Rule (Take $h=1$). Also find the value of the integral by actual integration. 7

- c) Using Euler's Method compute the value of $y(0.1)$ given $y' = x + \frac{1}{y}, y(0) = 1$ (Take $h = 0.025$) 7

- 9 a) Using Newton's Interpolation Formula find $f(1.2)$ and $f(2.0)$ from the table. 10

x	1	1.4	1.8	2.2
$y = f(x)$	3.49	4.82	5.96	6.50

- b) Using Runge - Kutta Method of 4th order, find $y(0.8)$ correct to four decimal places if $\frac{dy}{dx} = y - x^2$ given $y(0.6) = 1.7379$ (Take $h = 0.1$) 10