

Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
**FIFTH SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2017**

**Course Code: CS309**

**Course Name: GRAPH THEORY AND COMBINATORICS (CS)**

Max. Marks: 100

Duration: 3 Hours

**PART A**

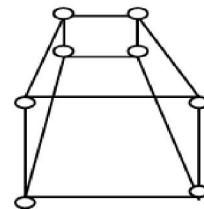
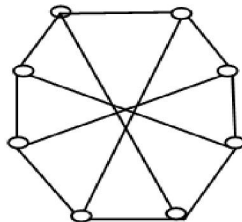
*Answer all questions, each carries 3 marks.*

- |   |  | Marks |
|---|--|-------|
| 1 | Consider a graph G with 4 vertices: $v_1, v_2, v_3$ and $v_4$ and the degrees of vertices are 3, 5, 2 and 1 respectively. Is it possible to construct such a graph G? If not, why? | (3)   |
| 2 | Draw a disconnected simple graph G1 with 10 vertices and 4 components and also calculate the maximum number of edges possible in G1.   | (3)   |
| 3 | State Dirac's theorem for hamiltonicity and why it is not a necessary condition for a simple graph to have a Hamiltonian circuit.  | (3)   |
| 4 | Differentiate between symmetric and asymmetric digraphs with examples and draw a complete symmetric digraph of four vertices.  | (3)   |

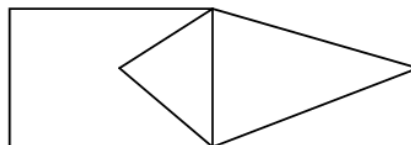
**PART B**

*Answer any two full questions, each carries 9 marks.*

- 5 a) What are the basic conditions to be satisfied for two graphs to be isomorphic? Are the two graphs below isomorphic? Explain with valid reasons (6)



- b) Write any two applications of graphs with sufficient explanation (3)
- 6 a) Consider the graph G given below: (4)



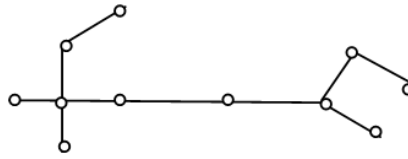
- Define Euler graph. Is G an Euler? If yes, write an Euler line from G.
- b) What is the necessary and sufficient condition for a graph to be Euler? And also prove it. (5)
- 7 a) Define Hamiltonian circuits and paths with examples. Find out the number of edge-disjoint Hamiltonian circuits possible in a complete graph with five vertices (5)
- b) State Travelling-Salesman Problem and how TSP solution is related with Hamiltonian Circuits? (4)

**PART C**

*Answer all questions, each carries 3 marks.*

- 8 List down any two properties of trees and also prove the theorem: *A graph is a tree if and only if it is a minimally connected.* (3)

- 9 Consider the tree T, given below (3)



- 10 Label the vertices of T appropriately and find the center and diameter of T. Prove the statement: (3)

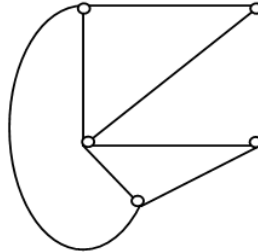
*Every cut-set in a connected graph G must also contain at least one branch of every spanning tree of G*

- 11 List down the properties stating the relationship between the edges of graph G and its dual  $G^*$  (3)

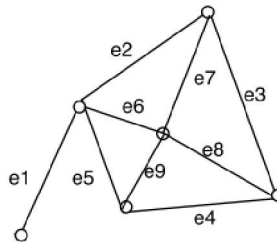
**PART D**

*Answer any two full questions, each carries 9 marks.*

- 12 a) Define spanning trees. Consider the graph G given below and obtain any *three* spanning trees from G. Calculate the number of distinct spanning trees possible from a complete graph with  $n$  vertices. (5)

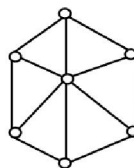


- b) Let  $G = (V, E)$  be a connected graph, and let  $T = (V, S)$  be a spanning tree of G. Let  $e = (a, b)$  be an edge of G *not in* T. Prove that, for any edge  $f$  on the path from  $a$  to  $b$  in T,  $(V, (S \cup \{e\}) - \{f\})$  is another spanning tree for G (4)
- 13 a) Define cut set. Find any four cut sets from the graph G given below and also find the edge connectivity of G. (5)



- b) Define vertex connectivity and draw a graph with an articulation point. (3)
- c) State Euler's Theorem (*formula*). (1)

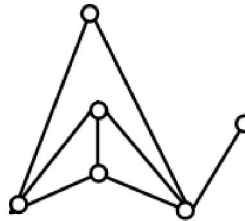
- 14 a) Draw two Kuratowski's graphs and also prove that Kuratowski's first graph is non planar using appropriate inequality. (4)
- b) Draw the geometric dual ( $G^*$ ) of the graph G given below and also check whether G and  $G^*$  are self dual or not, substantiate your answer clearly? (5)



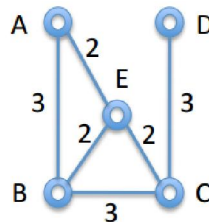
**PART E**

*Answer any four full questions, each carries 10 marks.*

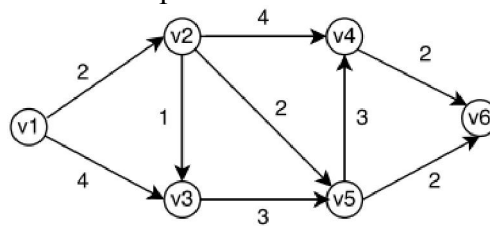
- 15 a) List down any four properties of adjacency matrix (4)
- b) Construct an adjacency matrix(X) for the following graph and also mention how the concept of edge sequences is described with  $X^3$  (no need to find  $X^3$  from X) (6)



- 16 a) Prove the theorem: (4)  
If  $A(G)$  is an incidence matrix of a connected graph  $G$  with  $n$  vertices, the rank of  $A(G)$  is  $n-1$
- b) Describe with examples the usage of incidence matrix to find two graphs ( $g_1$  and  $g_2$ ) are isomorphic. (6)
- 17 a) Define cut-set matrix with an example and list down any four properties of cut-set matrix (6)
- b) If  $B$  is a circuit matrix of a connected graph  $G$  with  $e$  edges and  $n$  vertices, then show that the number of linearly independent rows in  $B = e-n+1$  (4)
- 18 a) Draw the flow chart of minimum spanning-tree algorithm. (7)
- b) Find MST from the graph given below by simply applying Kruskal's procedure. (3)



- 19 Write the Dijkstra's shortest path algorithm (no need to draw flowchart). Apply this algorithm to find the shortest path between  $v_1$  and  $v_6$  (10)



- 20 Draw the flowchart of *Connectedness and Components* algorithm and also apply this algorithm on any graph (G) with 2 components. (10)

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**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
**FIFTH SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018**

**Course Code: CS309**

**Course Name: GRAPH THEORY AND COMBINATORICS**

Max. Marks: 100

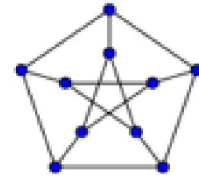
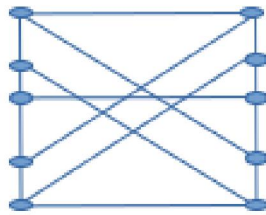
Duration: 3 Hours

**PART A**

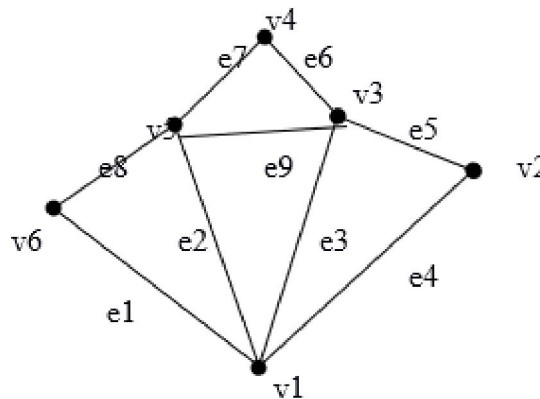
*Answer all questions, each carries 3 marks*

Marks

- 1 Define isomorphism between two graphs. Are the following graphs are isomorphic to each other? Justify your answer. (3)



- 2 For the following graph, find the shortest path between from  $v_1$  to  $v_4$ . Also find a Euler circuit. (3)



- 3 Define the following with example. (3)  
 i) Isomorphic digraph      ii) Complete symmetric digraph
- 4 Define Hamiltonian graph. Find an example of a non-Hamiltonian graph with a Hamiltonian path. (3)

**PART B**

*Answer any two full questions, each carries 9 marks*

- 5 a) For a Eulerian graph  $G$ , prove the following properties. (6)  
 i) The degree of each vertex of  $G$  is even. ii)  $G$  is an edge-disjoint union of cycles.
- b) Discuss the Konigsberg Bridge problem. Is there any solution to the problem? Justify your answer. (3)
- 6 a) Prove that a simple graph with  $n$  vertices must be connected, if it has more than  $(n-1)(n-2)/2$  edges. (6)
- b) 19 students in a nursery school play a game each day, where they hold hands to form a circle. For how many days can they do this, with no students holding hands with the same playmates more than once? Substantiate your answer with graph theoretic concepts. (3)

- 7 a) Prove that the number of odd degree vertices in a graph is always even. (4)  
 b) Show that in any group of two or more people, there are always two with exactly the same number of friends inside the group. (5)

### PART C

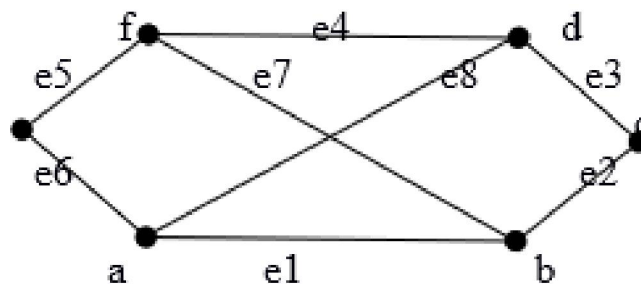
*Answer all questions, each carries 3 marks*

- 8 Discuss the dual of a subgraph with example. (3)  
 9 Write notes on the fundamental circuit. (3)  
 10 Prove that in a tree  $T(V,E), |V|=|E|+1$ . (3)  
 11 Define spanning tree with example. (3)

### PART D

*Answer any two full questions, each carries 9 marks*

- 12 Prove that the ring sum of any two cut-sets in a graph is either a third cut-set or an edge-disjoint union of cut-sets. (9)  
 13 a) Prove that a connected planar graph with  $n$  vertices and  $e$  edges has  $e-n+2$  regions. (4)  
 b) Consider the following graph  $G$  and any one of its spanning trees,  $T$ . List all fundamental circuits and fundamental cut-sets with respect to  $T$ . (5)



- 14 a) Show that the distance between vertices of a connected graph is a metric. (6)  
 b) Discuss the center of a tree with suitable example. (3)

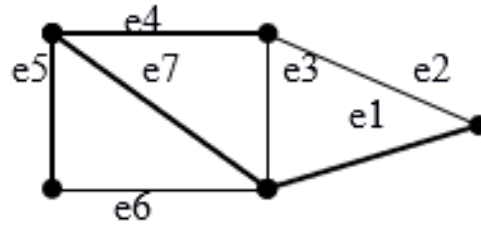
### PART E

*Answer any four full questions, each carries 10 marks*

- 15 a) Define the adjacency matrix  $X(G)$  of a graph. Let  $X(G)$  be adjacency matrix of a simple graph  $G$ , then prove that  $ij^{\text{th}}$  entry in  $X^r$  is the number of different edge sequences of  $r$  edges between vertices  $v_i$  and  $v_j$ . (6)  
 b) Draw the adjacency graph for the following adjacency matrix. (4)

$$X(G) = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

- 16 a) Define the circuit-matrix  $B(G)$  of a connected graph  $G$  with  $n$  vertices and  $e$  edges. Prove that the rank of  $B(G)$  is  $e-n+1$ . (6)  
 b) Write the fundamental circuit matrix with respect to the spanning tree shown in heavy lines for the following graph. (4)



- 17 Discuss an algorithm for finding the shortest path from a specified vertex to another specified vertex. Illustrate with example. (10)
- 18 Discuss an algorithm for finding the connected components of a graph G with suitable example. (10)
- 19 Discuss an algorithm to find the minimum spanning tree of a graph G with example. (10)
- 20 a) Define the incidence matrix of a graph G. Prove that the rank of an incidence matrix of a connected graph with n vertices is n-1. (6)
- b) Draw the graph represented by the following incidence matrix. (4)

$$X(G) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

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**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
**V SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2018**

**Course Code: CS309**

**Course Name: GRAPH THEORY AND COMBINATORICS**

Max. Marks: 100

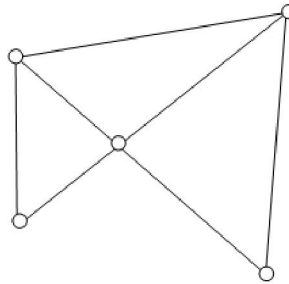
Duration: 3 Hours

**PART A**

*Answer all questions, each carries 3 marks.*

Marks

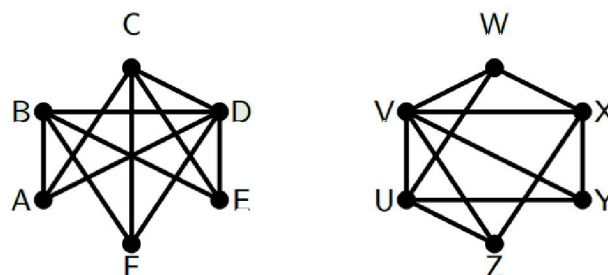
- |   |  |     |
|---|--|-----|
| 1 | Prove that the number of vertices of odd degree in a graph is always even  | (3) |
| 2 | Show that in a simple graph with $n$ vertices, the maximum number of edges is $n(n-1)/2$ and the maximum degree of any vertex is $n-1$ . | (3) |
| 3 | Differentiate between complete symmetric and complete asymmetric graph with an example each.   | (3) |
| 4 | State Dirac's Theorem and check its applicability in the following graph, G  | (3) |



**PART B**

*Answer any two full questions, each carries 9 marks.*

- 5 a) Define isomorphism between graphs? Are the two graphs below isomorphic? Justify (5)



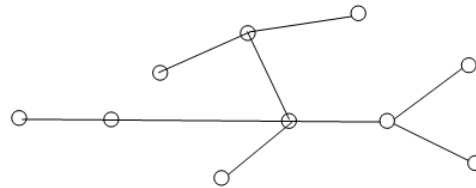
- b) Consider a complete graph  $G$  with 11 vertices. (4)
1. Find the maximum number of edges possible in  $G$ .
  2. Find the number of edge-disjoint Hamiltonian circuits in  $G$ .
- 6 a) A connected graph  $G$  is an Euler graph if and only if all vertices of  $G$  are of even degree. Prove the statement. (6)
- b) There are 37 telephones in the city of Istanbul, Turkey. Is it possible to connect them with wires so that each telephone is connected with exactly 7 others? Substantiate your answer with graph concepts. (3)
- 7 a) Give any two applications of graphs. Explain. (2)

- b) Define Hamiltonian circuit. Give an example. What general class of graphs is guaranteed to have a Hamiltonian circuit? Also draw a graph that has a Hamiltonian path but not a Hamiltonian circuit. (4)
- c) Prove that if a connected graph  $G$  is decomposed into two subgraphs  $g_1$  and  $g_2$ , there must be at least one vertex common between  $g_1$  and  $g_2$ . (3)

### PART C

*Answer all questions, each carries 3 marks.*

- 8 Prove that the distance between vertices of a connected graph is a metric. (3)
- 9 i) Find the eccentricity of all vertices in  $G$  given below and also mark the center of  $G$ . (3)

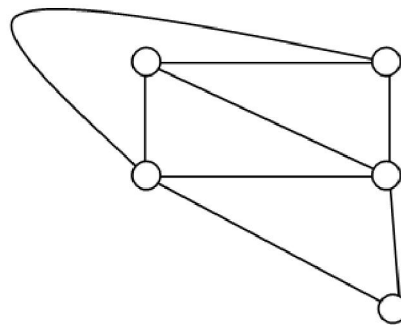


- ii) Find the number of possible labelled trees that can be constructed with 50 vertices. (3)
- 10 Draw the two simplest non-planar graphs and also mention their properties. (3)
- 11 What is the necessary and sufficient condition for two graphs to be duals of each other? Prove. (3)

### PART D

*Answer any two full questions, each carries 9 marks.*

- 12 a) Draw the geometric dual ( $G^*$ ) of  $G$  given and also write about the relationship between a planar graph  $G$  and its dual  $G^*$ . (6)



- b) Define rooted binary tree with an example. (3)
- 13 a) Find the number of edges and vertices of a graph  $G$  if its rank and nullity are 6 and 8 respectively. (2)
- b) Prove the statement, "Every circuit has an even number of edges in common with any cut-set". (4)
- c) Consider a binary tree with four weighted pendant vertices. Let their weights be 0.5, 0.12, 0.3 and 0.11. Construct a binary tree with minimum weighted path length. (3)
- 14 a) Define cut sets with an example. Give an application of finding cut-sets or edge. (4)

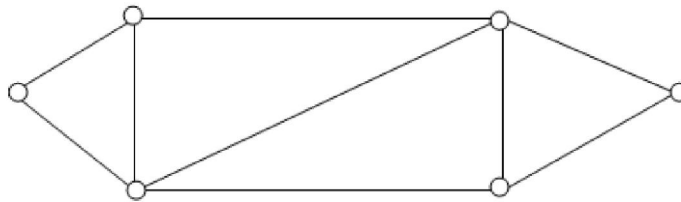
connectivity

- b) Define spanning tree. Show that the edges forming a spanning tree in a planar graph  $G$  correspond to the edges forming a set of chords in the dual  $G^*$  (5)

### PART E

*Answer any four full questions, each carries 10 marks.*

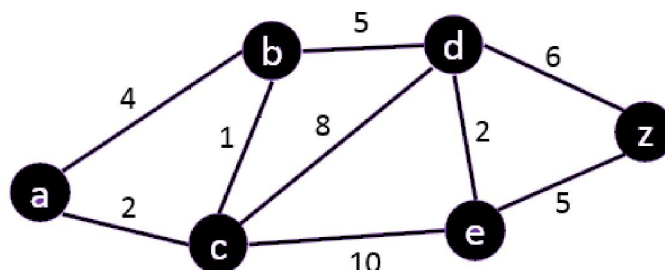
- 15 a) Draw the flow chart of spanning tree algorithm and also clearly mark the five conditions to be tested in connection with the spanning tree construction in the flowchart (6)
- b) Obtain a cut-set matrix for the following graph: (4)



- 16 a) Draw the flowchart to determine the components of a graph. (6)
- b) Define adjacency matrix and construct a graph from the following adjacency matrix: (4)

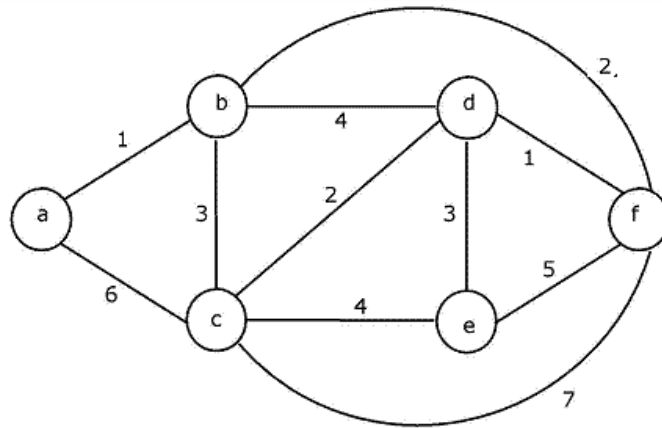
$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

- 17 a) Write edge listing and successor listing methods used in computer representation of graphs. (4)
- b) Two graphs  $G_1$  and  $G_2$  are isomorphic if and only if their incidence matrices  $A(G_1)$  and  $A(G_2)$  differ only by permutations of rows and columns (6)
- 18 a) Write the Dijkstra's Shortest Path Algorithm and apply this algorithm to find the shortest path between a and z (6)



- b) Let  $A$  and  $B$  be, respectively, the circuit matrix and incidence matrix of a self-loop-free graph  $G$ . Prove that  $A \times B^T = 0 \pmod{2}$  (4)
- 19 a) Define cut-set matrix and list down any four properties of cut-set matrix (5)
- b) Apply Kruskal's procedure to find the minimum spanning tree from the (5)

following graph G.



- 20 a) Prove that if  $B$  is a circuit matrix of a connected graph  $G$  with  $e$  edges and  $n$  vertices, then  $\text{rank of } B = e - n + 1$  (5)
- b) How can two linear arrays be used to represent a digraph. Give an example. (5)  
Compare this representation with edge list representation in terms of storage.

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**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
**V SEMESTER B.TECH DEGREE EXAMINATION(S), MAY 2019**

**Course Code: CS309**

**Course Name: GRAPH THEORY AND COMBINATORICS**

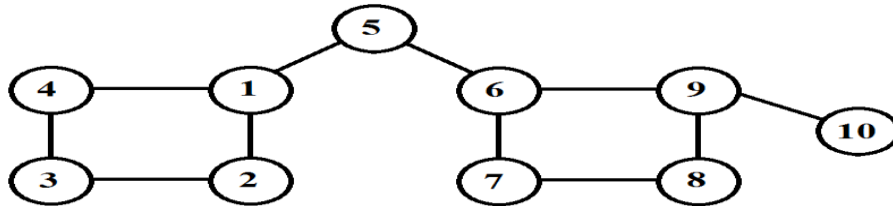
Max. Marks: 100

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**PART A**

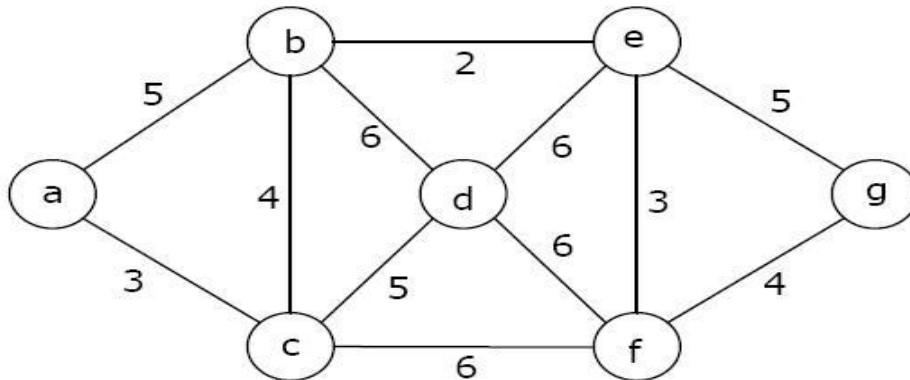
*Answer all questions, each carries 3 marks.*

- |   |   |   |
|---|---|---|
| 1 | Print a Walk, trail, path and cycle on the graph below. | 3 |
|---|---|---|



- |   |   |   |
|---|---|---|
| 2 | Define pendant vertex, isolated vertex and null graph with an example each. | 3 |
|---|---|---|

- |   |   |   |
|---|---|---|
| 3 | State travelling salesman problem. Print a travelling salesman's tour on the graph below. | 3 |
|---|---|---|

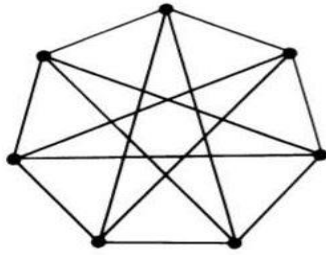


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|---|--|---|
| 4 | Prove Dirac's theorem for Hamiltonicity. | 3 |
|---|--|---|

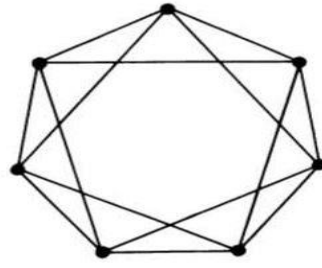
**PART B**

*Answer any two full questions, each carries 9 marks.*

- |   |   |   |
|---|---|---|
| 5 | a) Define isomorphism of graphs. Show that the graphs (a) and (b) are isomorphic. | 4 |
|---|---|---|

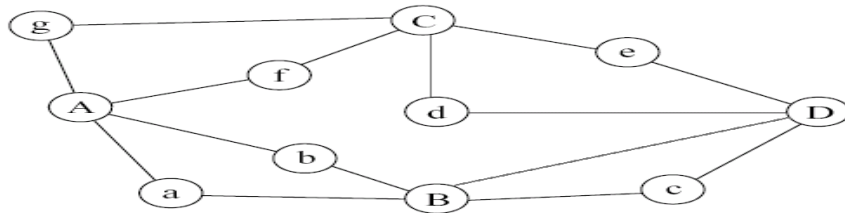


(a)

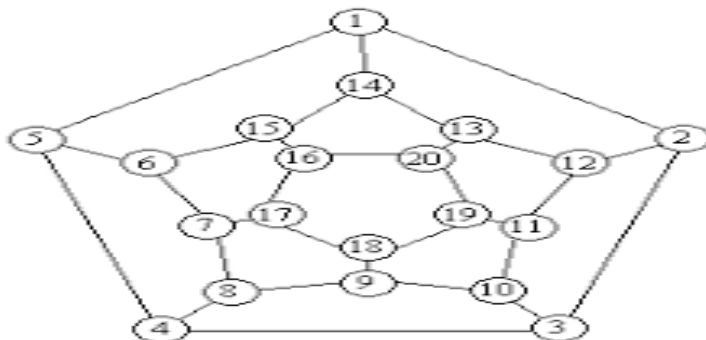


(b)

- b) Define subgraph. Give two subgraphs of the above graph.(Fig. a) 2
- c) Consider a complete graph G with 11 vertices. 3
  - 1. Find the maximum number of edges possible in G
  - 2. Find the number of edge-disjoint Hamiltonian circuits in G
- 6 a) Draw a simple disconnected graph with 10 vertices, 4 components and maximum 3  
number of edges.
- b) Explain any two applications of graphs. 2
- c) Check whether the given graph is an Euler graph and if yes, give the Euler line. 4  
Justify your answer.



- 7 a) Prove or disprove: If every vertex of a simple graph G has degree 2, then G is a 3  
cycle.
- b) Give Hamiltonian circuit of the following graph. 3

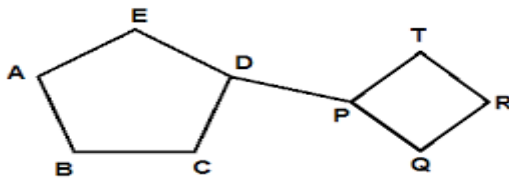


- c) In a graph G let  $p_1$  and  $p_2$  be two different paths between two given vertices. 3  
Prove that ringsum of  $p_1$  and  $p_2$  is a circuit or a set of circuits.

**PART C**

*Answer all questions, each carries 3 marks.*

- 8 Prove that in a graph  $G$ , if there is exactly one path between every pair of vertices, then  $G$  is a tree. 3
- 9 Given a spanning tree of a graph, how will you find out all spanning trees? 3
- 10 List all cut sets of the following graph. 3



- 11 Prove that every circuit has an even number of edges in common with any cut set. 3

**PART D**

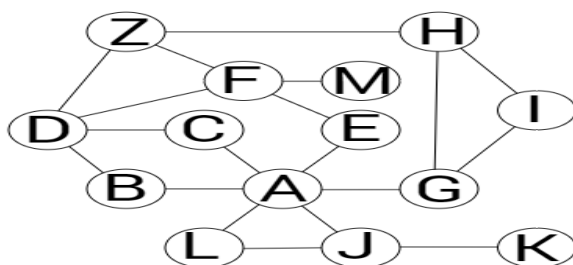
*Answer any two full questions, each carries 9 marks.*

- 12 a) Define a tree. Give any 4 properties of trees. 3
- b) Prove that a graph is a tree if and only if it is loop-less and has exactly one spanning tree. 3
- c) Prove that every circuit has an even number of edges in common with any cut set. 3
- 13 a) Prove that every tree has either one or two centers. 3
- b) Write short notes on geometric dual and combinatorial dual. 6
- 14 a) Draw a connected graph  $G$  and find two spanning trees  $T_1$  and  $T_2$  of  $G$  such that the distance  $(T_1, T_2) = 3$ . Find the branch set, chord set, rank and nullity of  $T_1$ . 4
- b) Construct a graph  $G$  with the following properties: Edge connectivity = 4, Vertex connectivity = 3 and degree of every vertex of  $G$  is greater than or equal to 5. 5

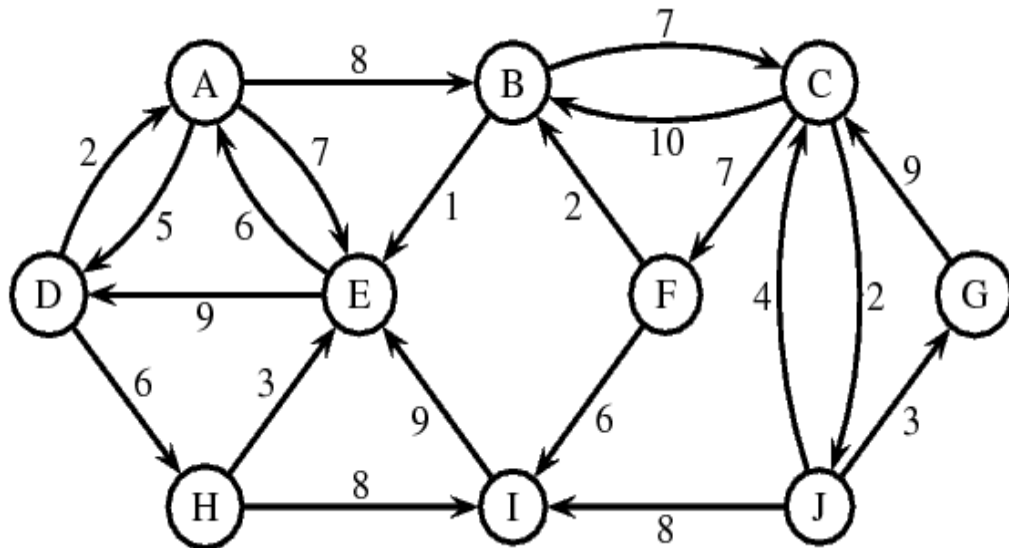
**PART E**

*Answer any four full questions, each carries 10 marks.*

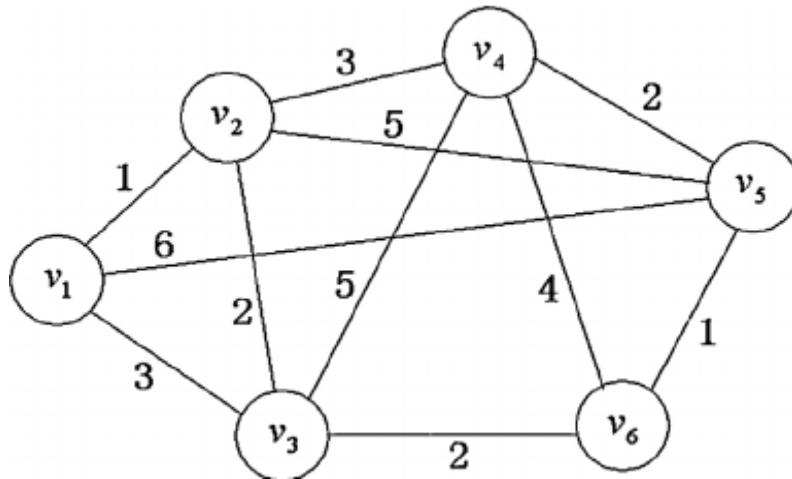
- 15 a) Give incidence matrix of the following graph. 3



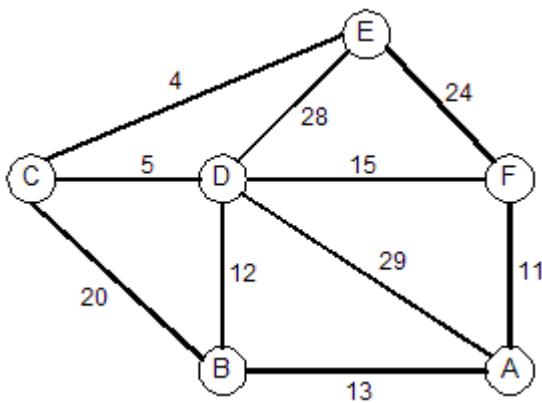
- b) Prove that two graphs  $G_1$  and  $G_2$  are isomorphic if and only if their incidence matrices  $A(G_1)$  and  $A(G_2)$  differ only by permutations of rows and columns. 2
- c) Give Dijkstra's algorithm to find shortest path between a vertex pair. Use it to find shortest path between A and G. 5



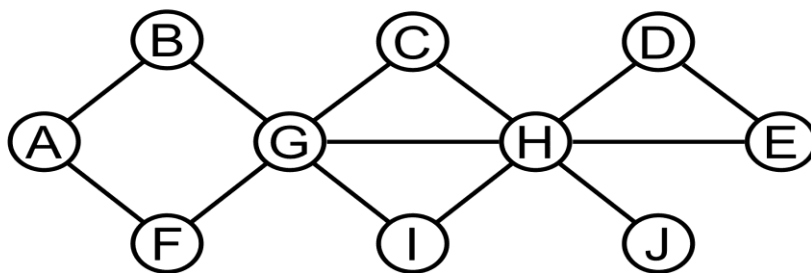
- 16 a) Prove that if  $B$  is a circuit matrix of a connected graph  $G$  with  $n$  vertices and  $e$  edges, then rank of  $B$  is  $e-n+1$ . 3
- b) How will you get fundamental circuit matrix from a circuit matrix. Derive the rank of a fundamental circuit matrix. 2
- c) Explain successor listing and incidence matrix methods used in computer representation of a graph? 5
- 17 a) Prove that the rank of cut set matrix  $C(G)$  is equal to rank of the incidence matrix  $A(G)$ , which equals the rank of the graph  $G$ . 3
- b) Define path matrix. What is the disadvantage of path matrix compared to other matrices. 2
- c) Find a minimum spanning tree of the following graph. Also give its rank and nullity. 5



- 18 a) If  $A(G)$  is an incidence matrix of a connected graph  $G$  with  $n$  vertices, then the rank of  $A(G)$  is  $n-1$ . 5
- b) How is Kruskal's algorithm used to find minimum cost spanning tree of a graph. Find a minimum spanning tree in the graph below. 5



- 19 a) Write cut set matrix of the following graph. Give its rank. 5



- b) Give an algorithm to check whether a graph is connected or not. How it can be implemented with an adjacency matrix. 5
- 20 a) Give any five properties of circuit matrix. 5
- b) How are edge listing and linear arrays used in computer representation of a graph? 5

Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
**FIFTH SEMESTER B.TECH DEGREE EXAMINATION(R&S), DECEMBER 2019**

**Course Code: CS309**

**Course Name: GRAPH THEORY AND COMBINATORICS**

Max. Marks: 100

Duration: 3 Hours

**PART A**

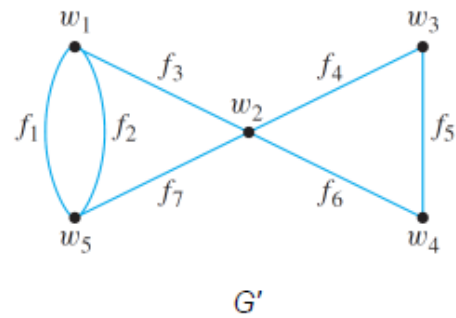
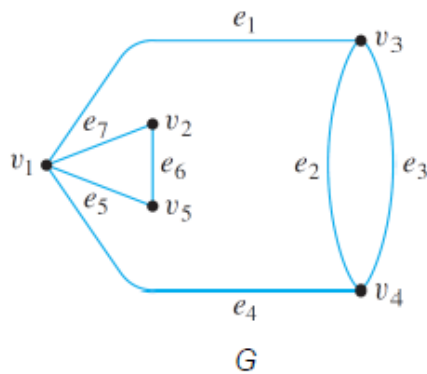
*Answer all questions, each carries 3 marks.*

- |   |   | Marks |
|---|---|-------|
| 1 | Define the terms a) Walk b) Path and c) Circuit with an example.  | (3)   |
| 2 | Prove that the no of vertices of odd degree in a graph is always even   | (3)   |
| 3 | Draw a graph that has a Hamiltonian path but does not have a Hamiltonian circuit.   | (3)   |
| 4 | Differentiate between Symmetric and Asymmetric digraphs with examples and draw a complete symmetric digraph of four vertices. | (3)   |

**PART B**

*Answer any two full questions, each carries 9 marks.*

- |   |   |     |
|---|---|-----|
| 5 | a) Prove that a simple graph with n vertices and k components can have at most $(n-k)(n-k+1)/2$ edges | (4) |
|   | b) Define Isomorphism of graphs. Check whether the two graphs are isomorphic or not                   | (5) |



- |   |   |     |
|---|---|-----|
| 6 | a) Define Euler graph. Check whether the graph is an euler graph or not. If yes, give the Euler line and justify your answer. | (5) |
|---|---|-----|



- |  |   |     |
|--|---|-----|
|  | b) Prove that a connected graph G is an Euler graph if and only if all vertices of G are of | (4) |
|--|---|-----|

even degree.

- 7 a) State travelling salesman problem and how TSP solution is related to Hamiltonian circuits. (5)
- b) State and Prove Dirac's Theorem for Hamiltonicity. (4)

### PART C

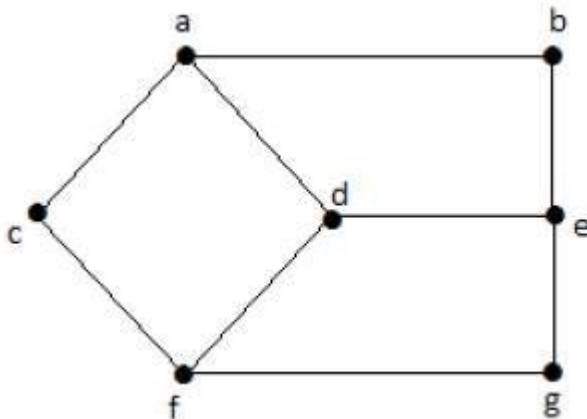
*Answer all questions, each carries 3 marks.*

- 8 Prove that the distance between the vertices of a connected graph is a metric (3)
- 9 List down any two properties of a tree and also prove the following theorem: A graph is a tree if and only if it is minimally connected. (3)
- 10 Define the terms vertex connectivity and edge connectivity with examples. (3)
- 11 Give the different representations of a planar graph. (3)

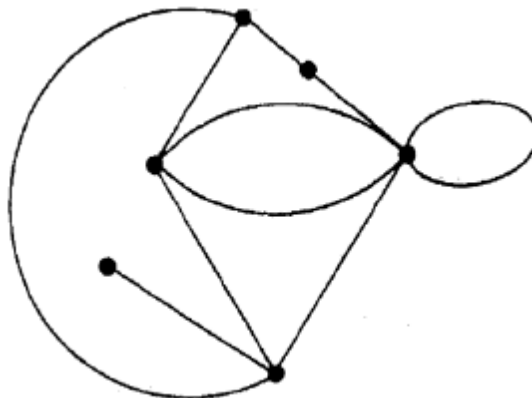
### PART D

*Answer any two full questions, each carries 9 marks.*

- 12 a) Find the eccentricity of all vertices in the graph G given below and also mark the center, radius and diameter of G (6)

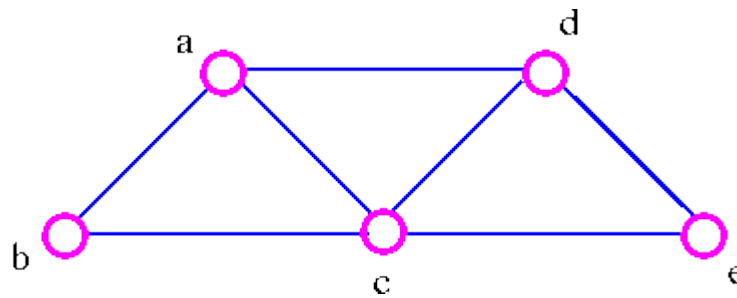


- b) State and prove Cayley's theorem (3)
- 13 a) Find the Geometrical dual ( $G^*$ ) of the graph G given below (5)



- b) List out the properties stating the relationship between the graph G and its dual  $G^*$  (4)

- 14 a) Consider the graph G and any one of its spanning tree T. Find all fundamental circuits and fundamental cut sets with respect to the spanning tree T. (6)



- b) Prove that “Every cut set in a connected graph G must contain atleast one branch of every spanning tree of G”. (3)

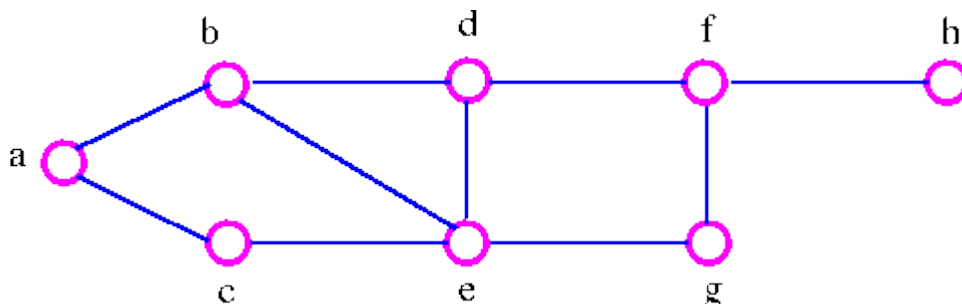
### PART E

*Answer any four full questions, each carries 10 marks.*

- 15 a) Define Adjacency Matrix  $X(G)$  of a graph. Determine the properties of adjacency matrix. (6)
- b) Draw the graph represented by the following adjacency matrix. (4)

$$X(G) = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

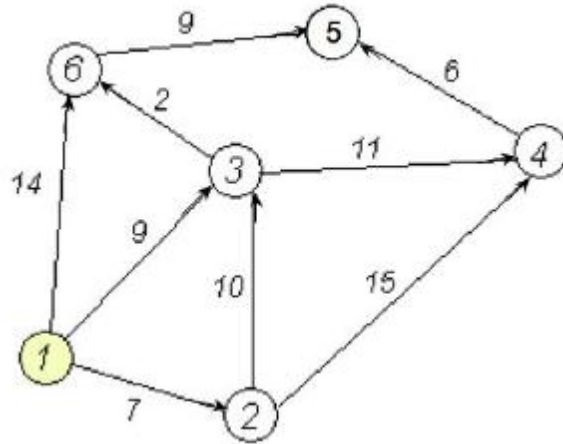
- 16 a) Obtain a cut set matrix for the following graph (6)



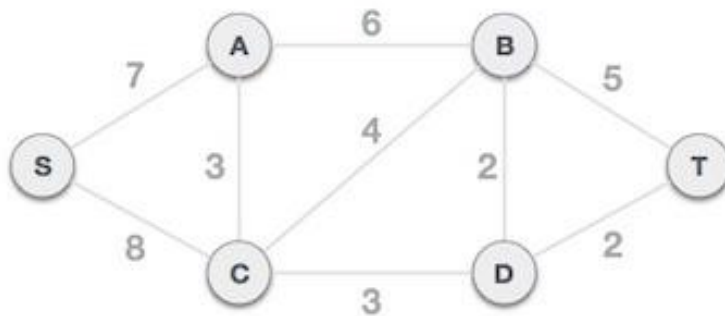
- b) Define path matrix. Determine the properties of a path matrix. (4)
- 17 a) Explain edge listing and successor listing methods used in computer representation of graphs (4)
- b) Draw the flow chart to determine connectedness and components of a graph (6)
- 18 Draw a flowchart indicating all the five conditions to find the spanning tree /spanning (10)

forest. Apply it to find the spanning tree /spanning forest for any graph of your choice.

- 19 Write Dijkstra's Shortest path algorithm and apply this algorithm to find the shortest path (10)  
path



- 20 Write Kruskal's algorithm to find the minimum spanning tree of a graph G. Apply it (10)  
to find the MST for the graph given below



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Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**

Fifth semester B.Tech degree examinations (S) September 2020

**Course Code: CS309****Course Name: GRAPH THEORY AND COMBINATORICS**

Max. Marks: 100

Duration: 3 Hours

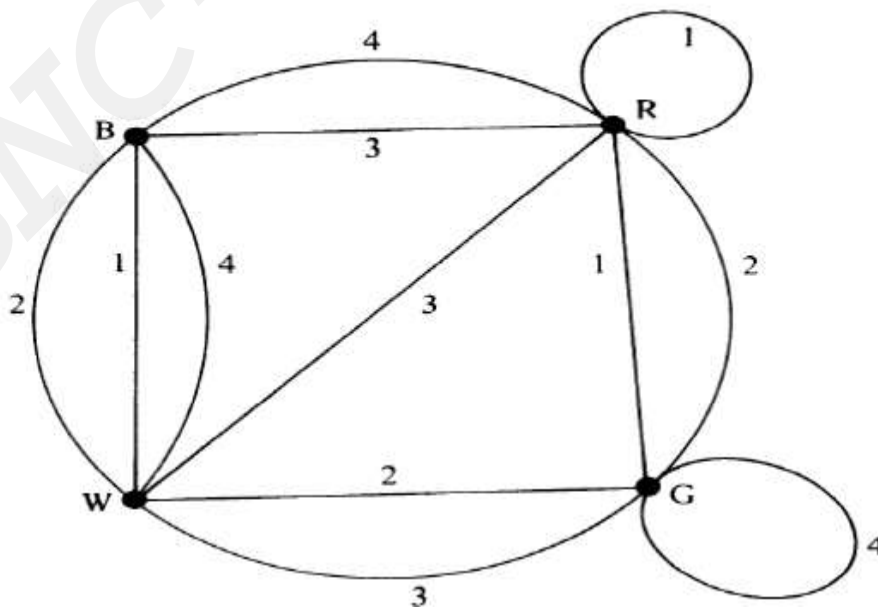
**PART A***Answer all questions, each carries 3 marks.*

Marks

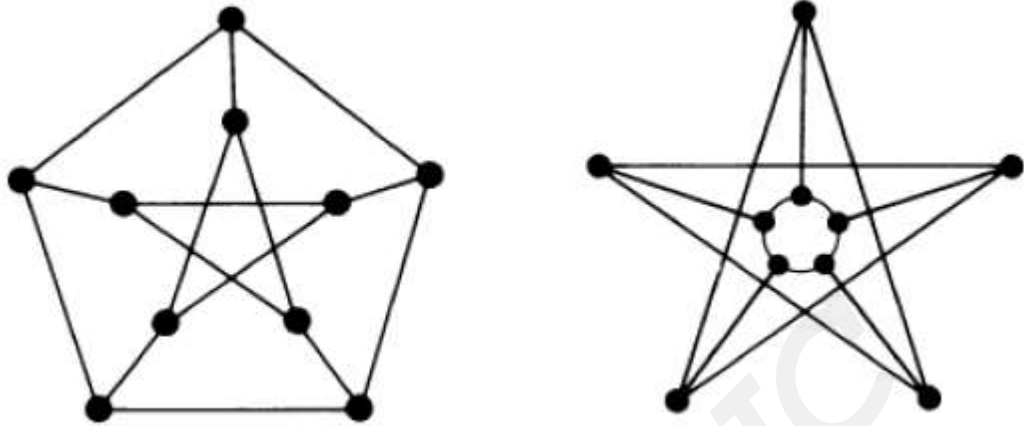
- |   |  |     |
|---|--|-----|
| 1 | Define pendent vertex, isolated vertex and null graph with an example.   | (3) |
| 2 | Show that in a simple graph with n vertices ,the maximum number of edges is $n(n-1)/2$   | (3) |
| 3 | Define Hamiltonian circuits and path with examples. Find out the number of edge disjoint Hamiltonian circuits possible in a complete graph with five vertices. | (3) |
| 4 | State and prove Dirac's Theorem of Hamiltonicity.  | (3) |

**PART B***Answer any two full questions, each carries 9 marks.*

- |   |  |     |
|---|--|-----|
| 5 | a) Define subgraphs. What are edge disjoint and vertex disjoint subgraphs? Construct two edge disjoint subgraphs of the graph G. | (4) |
|---|--|-----|



- b) Check whether the two graphs are isomorphic or not. Justify your answer. (5)

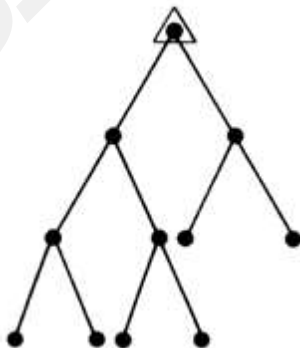


- 6 a) Prove that in a complete graph with  $n$  vertices there are  $(n-1)/2$  edge disjoint Hamiltonian circuits, if  $n$  is an odd number  $\geq 3$  (5)
- b) Explain arbitrarily traceable graphs with suitable examples. (4)
- 7 a) Is it possible to have simple graphs with the following degree sequences?if yes,draw the graphs (5)
- a) 2,3,3,3,3,3,4,5
  - b) 1,3,3,4,5,6,6
  - c) 1,2,3,3,4,5,6
- b) Explain digraphs and binary relation on digraphs. (4)

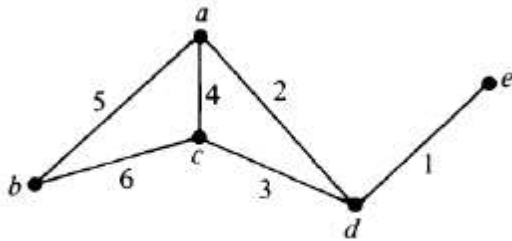
**PART C**

*Answer all questions, each carries 3 marks.*

- 8 Prove that in a graph  $G$ , if there is exactly one path between every pair of vertices, then  $G$  is a tree. (3)
- 9 Define rooted binary tree. Find the path length of the following tree (3)



- 10 Sketch all spanning trees of the given graph (3)

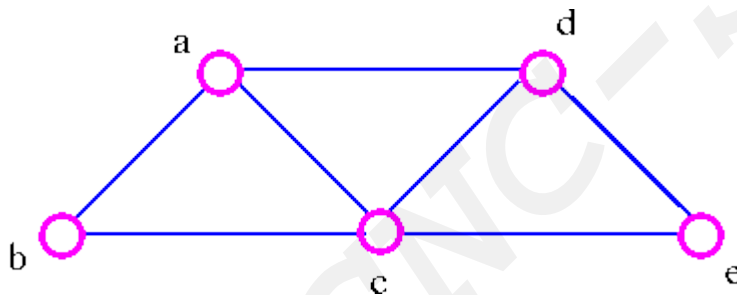


- 11 Draw the two simplest non planar graphs and also mention their properties. (3)

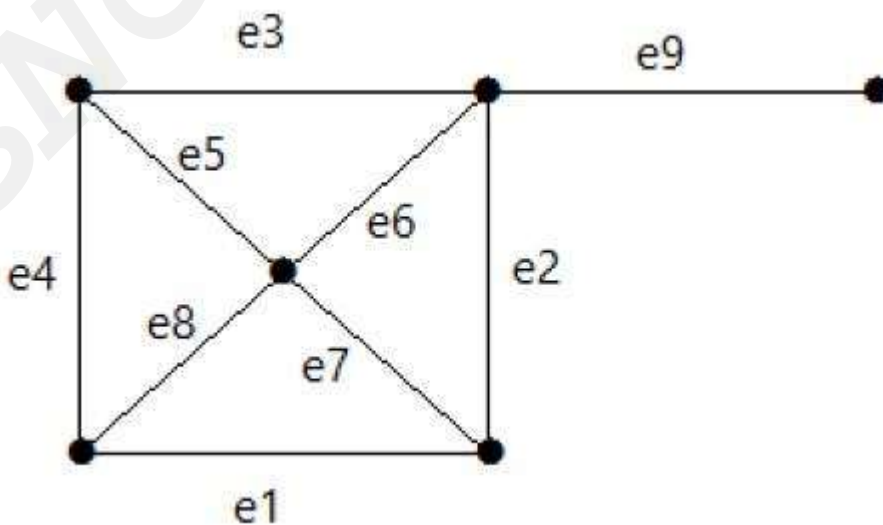
**PART D**

*Answer any two full questions, each carries 9 marks.*

- 12 a) Define Spanning tree. Find any two spanning trees  $T_1$ ,  $T_2$  of the graph  $G$  given below. Also find the branch set, chord set, rank and nullity. (6)

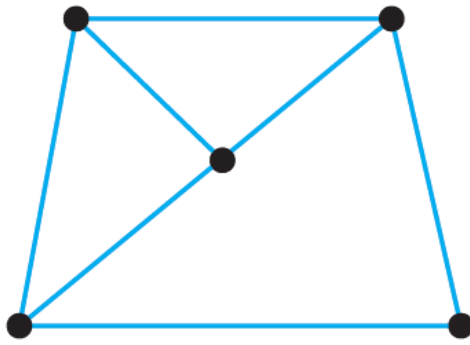


- b) Sketch two different binary trees on 13 vertices, one having maximum height and other having minimum height. (3)
- 13 a) Define Cut set .Find all cutsets of the graph  $G$  given below and also find the edge connectivity of  $G$ . (6)



- b) Define vertex connectivity and draw a graph with an articulation point. (3)

- 14 a) Draw the geometric dual of the graph G given below. (4)

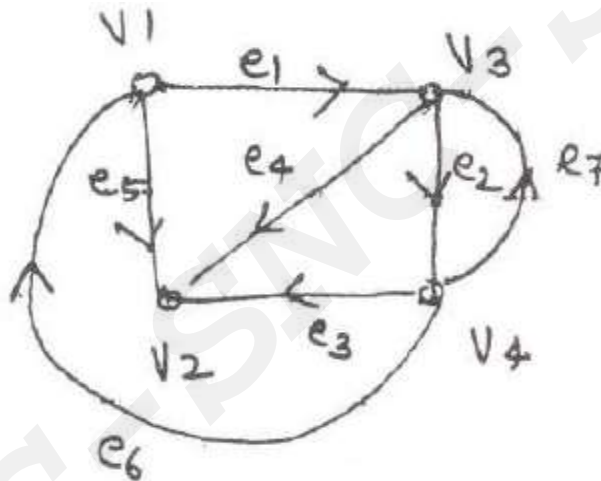


- b) Prove that a connected planar graph with  $n$  vertices and  $e$  edges has  $e-n+2$  regions. (5)

**PART E**

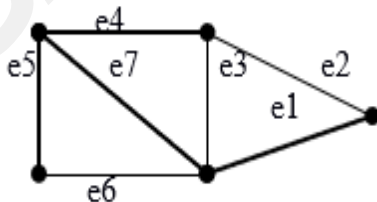
*Answer any four full questions, each carries 10 marks.*

- 15 a) Give the incidence matrix of the graph G. Also write the properties of incidence matrix. (6)



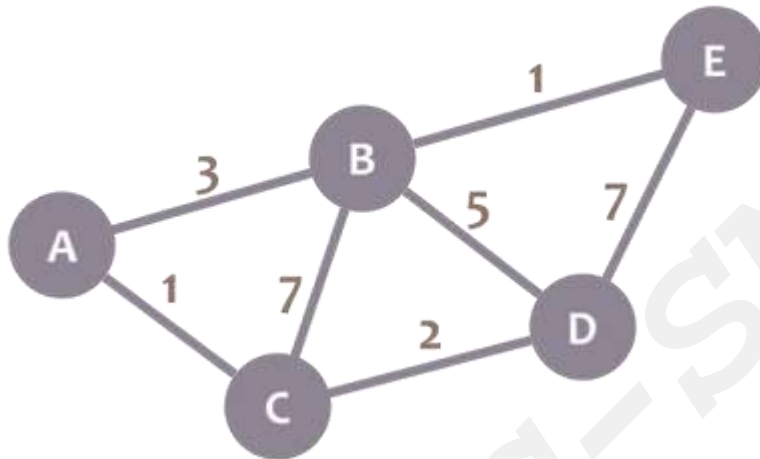
matrix. . . . .

- b) Prove that the rank of the incidence matrix of a connected graph G is  $n-1$ . (4)
- 16 a) Find the Fundamental Circuit matrix of the give graph G with respect to the spanning tree shown in heavy lines. Also find its rank. (6)



- b) Prove that "If B is a circuit matrix of a connected graph G with  $e$  edges and  $n$  vertices then  $\text{rank of } B = e - n + 1$ " (4)
- 17 a) Explain different methods used in computer representation of graphs with an example. (5)
- b) Draw the flow chart to determine connectedness and components of a graph. (5)

- 18 a) Draw a flowchart indicating all the five conditions to find the spanning tree /spanning forest. Apply it to find the spanning tree /spanning forest for any disconnected graph of your choice. (10)
- 19 a) Write Dijkstra's Shortest path algorithm and apply this algorithm to find the shortest path (10)



- 20 a) Discuss an algorithm to find the minimum spanning tree of a graph G with an example (10)

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