

Reg. No. _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
THIRD SEMESTER B.TECH DEGREE EXAMINATION, JANUARY 2017

Course Code: **CS 201**

Course Name: **DISCRETE COMPUTATIONAL STRUCTURES (CS, IT)**

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions. Each Question carries 3 marks

1. Show that $(A-B) - C = A - (B \cup C)$
2. Show that the set of integers of positive, negative and zero are denumerable.
3. Show that if any five integers from 1 to 8 are chosen, then atleast two of them will have a sum 9.
4. Define: Partition, antisymmetric, Semigroup homomorphism.

PART B

Answer any two questions. Each Question carries 9 marks.

5. a. Prove that every equivalence relation on a set generates a unique partition of the set and the blocks of this partition corresponds to R-equivalence classes. (4.5)
b. Let $X = \{1, 2, \dots, 7\}$ and $R = \{ \langle X, Y \rangle / X-Y \text{ is divisible by } 3 \}$. Show that R is an equivalence relation. Draw the graph R. (4.5)
6. a. In how many ways can the letters of the word MONDAY be arranged? How many of them begin with M and end with Y? How many of them do not begin with M but end with Y? (4)
b. Solve $a_{n+2} - 4a_{n+1} + 4a_n = 2^n$, $a_0=1$, $a_1=1$ (5)
7. a. Draw Hasse diagram for D_{100} . Find GLB and LUB for $B = \{10, 20\}$, $B = \{5, 10, 20, 25\}$ (3)
b. Let $X = \{1, 2, 3\}$ and f, g, h be function from X to X given by $f = \{(1, 2), (2, 3), (3, 1)\}$
 $g = \{(1, 2), (2, 1), (3, 3)\}$ $h = \{(1, 1), (2, 2), (3, 1)\}$. Find $f \circ g$, $g \circ h$, $f \circ h \circ g$. (3)
c. In a class of 25 students, 12 have taken Mathematics, 8 have taken Mathematics but not Biology. Find the number of students who have taken Mathematics and Biology and those who have taken Biology but not Mathematics. (3)

PART C

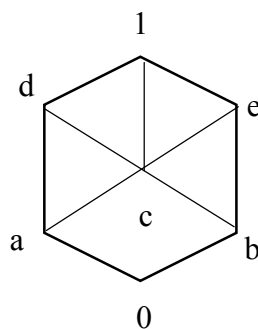
Answer All Questions. Each Question carries 3 marks.

8. Show that inverse of an element a in the group is unique.
9. Show that $(G, +_6)$ is acyclic group where $G = \{0, 1, 2, 3, 4, 5\}$
10. $A = \{2, 3, 4, 6, 12, 18, 24, 36\}$ with partial order of divisibility. Determine the POSET is a lattice.
11. Consider the lattice D_{20} and D_{30} of all positive integer divisors of 20 and 30 respectively, under the partial order of divisibility. Show that is a Boolean algebra.

PART D

Answer any two Questions. Each Question carries 9 marks

12. a. Prove that the order of each subgroup of a finite group G is a divisor of the order of the group G . (4.5)
 b. Show that the set $\{0, 1, 2, 3, 4, 5\}$ is a group under addition and multiplication modulo 6. (4.5)
13. a. Prove that every finite integral domain is a field. (4.5)
 b. Show that (Z, θ, Θ) is a ring where $a \theta b = a+b-1$ and $a \Theta b = a+b-ab$ (4.5)
14. a. Consider the Boolean algebra D_{30} . Determine the following:
 i) All the Boolean sub-algebra of D_{30} .
 ii) All Boolean algebras which are not Boolean sub-algebras of D_{30} having atleast four elements. (4.5)
 b. Consider the Lattice L in the figure. Find the L is distributive and complemented lattice. Also find the complement of a, b, c . (4.5)



PART E

Answer any four Questions. Each Question carries 10 marks

15. a. Without using truth tables, prove the following $(\neg P \vee Q) \wedge (P \wedge (P \wedge Q)) \equiv P \wedge Q$
 b. Show that $((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$ is a tautology.
16. a. Convert the given formula to an equivalent form which contains the connectives \neg and \wedge only: $\neg (P \leftrightarrow (Q \rightarrow (R \vee P)))$
 b. Show that $\neg P \wedge (\neg Q \wedge R) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$.
17. a. Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$
 b. Prove the validity of the following argument “If I get the job and work hard, then I will get promoted. If I get promoted, then I will be happy. I will not be happy. Therefore, either I will not get the job or I will not work hard.
18. a. Prove that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)(P(x) \wedge (\exists x)(Q(x)))$
 b. Consider the statement “Given any positive integer, there is a greater positive integer”. Symbolize this statement with and without using the set of positive integers as the universe of discourse.
19. a. Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$ and $\neg M$.
 b. Prove by mathematical induction that $6^{n+2} + 7^{2n+1}$ is divisible by 43 for each positive integer n.
20. Discuss indirect method of Proof. Show that the following premises are inconsistent.
 (i) If Jack misses many classes through illness, then he fails high school.
 (ii) If Jack fails high School, then he is uneducated.
 (iii) If Jack reads a lot of books, then he is not uneducated.
 (iv) Jack misses many classes through illness and reads a lot of books.

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
THIRD SEMESTER BTECH DEGREE EXAMINATION JULY 2017

Course Code: **CS 201**

Course Name: **DISCRETE COMPUTATIONAL STRUCTURES (CS, IT)**

Max Marks: 100

Duration: 3 hrs

PART A

Answer all questions, 3 marks each.

1. Draw the Hasse diagram for the divisibility relation on the set $A = \{2,3,6,12,24,36\}$
2. Define equivalence relation? Give an example of a relation that is not an equivalence relation?
3. In how many different ways can the letters of the word 'MATHEMATICS' be arranged such that the vowels must always come together?
4. Define homomorphism and isomorphism?

PART B

Answer any 2 Questions, 9 marks each.

5. (a) Consider f, g and h are functions on integers $f(n) = n^2$, $g(n) = n + 1$, $h(n) = n - 1$. Determine
 i) $f \circ g \circ h$ ii) $g \circ f \circ h$ iii) $h \circ f \circ g$ (6)
- (b) State Pigeonhole Principle. Prove that at least two of the children were born on the same day of the week. (3)
6. (a) Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + (n+1)2^n$ (6)
 (b) Show that set of all integers is countable (3)
7. (a) Find the number of integers between 1 and 250 both inclusive that are not divisible by any of integers 2,3,5 and 7. (5)
 (b) If $*$ is the binary operation on the set R of real numbers defined by $a*b = a + b + 2ab$. Find if $\{R, *\}$ is a semigroup. Is it commutative? (4)

PART C

Answer all questions, 3 marks each.

8. Show that the set $\{1,2,3,4,5\}$ is not a group under addition modulo 6.
9. Prove that every distributive lattice is modular.
10. Show that the set Q^+ of all positive rational numbers forms an abelian group under the operation $*$ defined by $a*b = \frac{1}{2}ab$; $a, b \in Q^+$
11. Simplify the Boolean expression $a'b'c + ab'c + a'b'c'$

PART D

Answer any 2 Questions, 9 marks each.

12. For the set $I_4 = \{0, 1, 2, 3\}$, show that modulo 4 system is a ring. (9)
13. a) Prove that the order of each sub group of a finite group G is a divisor of the order of group G . (6)
- b) If $A = (1\ 2\ 3\ 4\ 5)$ and $B = (2\ 3)(4\ 5)$. Find product of permutation AB . (3)
14. If (L, \leq) is a lattice, then for any $a, b, c \in L$, the following properties hold. If $b \leq c$, then i) $a \vee b \leq a \vee c$ ii) $a \wedge b \leq a \wedge c$ (9)

PART E

Answer any 4 Questions, 10 marks each.

15. (a) Show that $(P \rightarrow Q) \wedge (Q \rightarrow P)$ is logically equivalent to $P \leftrightarrow Q$. (5)
- (b) Suppose x is a real number. Consider the statement "If $x^2 = 4$, then $x = 2$." Construct the converse, inverse, and contrapositive (5)
16. (a) Prove that $(P \wedge Q) \rightarrow (P \leftrightarrow Q)$ is a tautology. (5)
- (b) Show that $(a \vee b)$ follows logically from the premises $p \vee q, (p \vee q) \rightarrow \neg r, \neg r \rightarrow (s \wedge \neg t)$ and $(s \wedge \neg t) \rightarrow (a \vee b)$ (5)
17. (a) Represent the following sentence in predicate logic using quantifiers i) All men are mortal. ii) Every apple is red iii) Any integer is either positive or negative. (6)
- (b) Use the truth table to determine whether $p \rightarrow (q \wedge r)$ and $(p \wedge r) \rightarrow q$ are logically equivalent. (4)
18. Show that the premises "one student in this class knows how to write programs in JAVA" and "Everyone who knows how to write programs in JAVA can get a high paying job" imply the conclusion "Someone in this class can get a high paying job" (10)
19. (a) Prove the following statement by contraposition:
If a product of two positive real numbers is greater than 100, then at least one of the numbers is greater than 10. (6)
- (b) Express the negation of the following statement in English using quantifiers
i) If the teacher is absent, then some students do not keep quiet ii) All students keep quiet and teacher is present. (4)
20. (a) Prove that $\sqrt{2}$ is irrational using proof by contradiction. (6)
- (b) Find the truth table $(\sim Q \Rightarrow \sim P) \Rightarrow (P \Rightarrow Q)$ (4)

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
THIRD SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2017

Course Code: CS201

Course Name: DISCRETE COMPUTATIONAL STRUCTURES (CS, IT)

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks.

Marks

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|---|--|-----|
| 1 | Assume $A = \{1, 2, 3\}$ and $\rho(A)$ be its power set. Let \subseteq be the subset relation on the power set. Draw the Hasse diagram of $(\rho(A), \subseteq)$ | (3) |
| 2 | Let R denote a relation on the set of ordered pairs of positive integers such that $(x, y)R(u, v)$ iff $xv = yu$. Show that R is an equivalence relation | (3) |
| 3 | Prove that in any group of six people, at least three must be mutual friends or at least three must be mutual strangers. | (3) |
| 4 | Define GLB and LUB for a partially ordered set. Give an example | (3) |

PART B

Answer any two full questions, each carries 9 marks.

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|---|--|-----|
| 5 | a) Suppose $f(x) = x + 2, g(x) = x - 2$ and $h(x) = 3x$ for $x \in \mathbb{R}$, where \mathbb{R} is the set of real numbers. Find $g \circ f, f \circ g, f \circ f, g \circ g, f \circ h, h \circ g, h \circ h$ and $(f \circ h) \circ g$ | (4) |
| | b) Prove that every equivalence relation on a set generates a unique partition of the set with the blocks as R -equivalence classes | (5) |
| 6 | a) Show that the set \mathbb{N} of natural numbers is a semigroup under the operation $x * y = \max(x, y)$. Is it a monoid? | (3) |
| | b) Solve the recurrence relation $a_r + 5a_{r-1} + 6a_{r-2} = 3r^2 - 2r + 1$ | (6) |
| 7 | a) Show that for any commutative monoid $\langle M, * \rangle$, the set of idempotent elements of M forms a submonoid. | (5) |
| | b) Define subsemigroups and submonoids. | (4) |

PART C

Answer all questions, each carries 3 marks.

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|----|--|-----|
| 8 | Show that, for an abelian group, $(a * b)^{-1} = a^{-1} * b^{-1}$ | (3) |
| 9 | Show that every chain is a distributive lattice. | (3) |
| 10 | Simplify the Boolean expression $a'b'c + ab'c + a'b'c'$ | (3) |
| 11 | Let $G = \{1, a, a^2, a^3\}$ ($a^4 = 1$) be a group and $H = \{1, a^2\}$ is a subgroup of G under multiplication. Find all cosets of H . | (3) |

PART D

Answer any two full questions, each carries 9 marks.

- 12 a) Show that the order of a subgroup of a finite group divides the order of the group. (6)
 b) Define ring homomorphism. (3)
- 13 Show that (I, \oplus, \otimes) is a commutative ring with identity, where the operations \oplus and \otimes are defined, for any $a, b \in I$, as $a \oplus b = a + b - 1$ and $a \otimes b = a + b - ab$. (9)
- 14 a) Let (L, \leq) be a lattice and $a, b, c, d \in L$. Prove that if $a \leq c$ and $b \leq d$, then (5)
 (i) $a \vee b \leq c \vee d$
 (ii) $a \wedge b \leq c \wedge d$
 b) Show that in a Boolean algebra, for any a, b, c (4)
 $(a \wedge b \wedge c) \vee (b \wedge c) = b \wedge c$

PART E

Answer any four full questions, each carries 10 marks.

- 15 a) a) Construct truth table for $(\sim p \wedge (\sim q \wedge r)) \vee ((q \wedge r) \vee (p \wedge r))$ (6)
 b) Explain proof by Contrapositive with example. (4)
- 16 Prove the following implication (10)
 $(x)(P(x) \vee Q(x)) \implies (x) P(x) \wedge (\exists x) Q(x)$
- 17 a) Represent the following sentences in predicate logic using quantifiers (6)
 (i) "x is the father of the mother of y"
 (ii) "Everybody loves a lover"
 b) Determine whether the conclusion C follows logically from the premises (4)
 $H_1: \sim p \vee q, H_2: \sim(q \wedge \sim r), H_3: \sim r$ C: $\sim p$
- 18 a) Without using truth table prove $p \rightarrow (q \rightarrow p) \iff \sim p \rightarrow (p \rightarrow q)$ (4)
 b) Determine the validity of the following statements using rule CP. (6)
 "my father praises me only if I can be proud of myself. Either I do well in sports or I can't be proud of myself. If I study hard, then I can't do well in sports. Therefore if my father praises me then I do not study well"
- 19 a) Show that $r \rightarrow s$ can be derived from the premises $p \rightarrow (q \rightarrow s), \sim r \vee p, q$ (4)
 b) Prove, by Mathematical Induction, that $n(n+1)(n+2)(n+3)$ is divisible by 24, for all natural numbers n (6)
- 20 a) "If there are meeting, then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time. There was no meeting". Show that these statements constitute a valid argument. (6)
 b) Show that $2^n < n!$ For $n \geq 4$ (4)

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
THIRD SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2018

Course Code: CS201

Course Name: DISCRETE COMPUTATIONAL STRUCTURES

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks.

Marks

- | | | |
|---|--|-----|
| 1 | What non zero entries are there in the relation matrix of intersection of R and its converse if R is an anti-symmetric relation? | (3) |
| 2 | Define a recurrence relation. Give an example | (3) |
| 3 | Show that the set of idempotent elements of any commutative monoid forms a submonoid. | (3) |
| 4 | Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = x^3 - 2$, where \mathbb{R} is the set of real numbers. Find f^{-1} | (3) |

PART B

Answer any two full questions, each carries 9 marks.

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|---|--|-----|
| 5 | a) Draw the Hasse diagram for the following sets under the partial ordering relation “Divides”, and indicates those which are totally ordered.
$\{2,6,24\}, \{1,2,3,6,12\}, \{2,4,8,16\}, \{3,9,27,54\}$ | (5) |
| | b) Prove that every equivalence relation on a set generates a unique partition of the set with the blocks as R-equivalence classes. | (4) |
| 6 | a) Solve the recurrence relation $T(k) - 7T(k-1) + 10T(k-2) = 6 + 8k$ with $T(0) = 1$ and $T(1) = 2$. | (5) |
| | b) What is the minimum number of students required in an English class to be sure that at least six will receive the same grade, if there are five possible grades? | (4) |
| 7 | a) From a group of 7 men and 6 women, 5 people are to be selected to form a committee, such that at least 3 men are there in the committee. In how many ways can the committee be formed? | (4) |
| | b) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$, where \mathbb{R} is the set of real numbers. Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 - 2$ and $g(x) = x + 4$. State whether these functions are injective, surjective, and bijective. | (5) |

PART C

Answer all questions, each carries 3 marks.

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|----|--|-----|
| 8 | How many proper subgroups will be there for a group of order 11? Justify your answer. | (3) |
| 9 | Show that every chain is a distributive lattice. | (3) |
| 10 | What conditions to be satisfied if an algebraic system $(A, +, \cdot)$ is called a ring? | (3) |
| 11 | Define a complemented lattice. Give an example | (3) |

PART D

Answer any two full questions, each carries 9 marks.

- | | | |
|----|--|-----|
| 12 | a) For a cyclic group of order n generated by an element a, show that n is the least | (5) |
|----|--|-----|

- positive integer for which $a^n = e$. (e is the identity element)
- b) Show that every finite integral domain is a field. (4)
- 13 a) Let $(L, *, \oplus)$ be any distributive lattice. For any a, b, c element of L, show that $(a*b = a*c)$ and $(a \oplus b = a \oplus c) \implies b = c$. (4)
- b) Define a Boolean algebra. Illustrate a two element Boolean Algebra with an example (5)
- 14 a) Prove that every finite group of order n is isomorphic to a permutation group of degree n. (5)
- b) Define Lattice homomorphism and direct product of two lattices. (4)

PART E

Answer any four full questions, each carries 10 marks.

- 15 a) Without using truth tables prove that $\neg(P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q)) \Leftrightarrow (\neg P \vee Q)$ (5)
- b) Using truth table determine whether the conclusion $\neg P$ follows logically from the premises $P \rightarrow Q$ and $\neg(P \wedge Q)$. (5)
- 16 a) Show that $R \vee S$ follows logically from the premises $C \vee D$, $(C \vee D) \rightarrow \neg H$, $\neg H \rightarrow (A \wedge \neg B)$, and $(A \wedge \neg B) \rightarrow (R \vee S)$. (4)
- b) If there was a ball game, then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time. Therefore, there was no ball game. Show that these statements constitute a valid argument. (6)
- 17 a) Symbolise the statement “x is the father of the mother of y” (4)
- b) Indicate the variables that are free and bound (6)
- i. $(x)(P(x) \wedge R(x)) \rightarrow (x)P(x) \wedge R(x)$
- ii. $(x)(P(x) \wedge (\exists x) Q(x)) \vee ((x)P(x) \rightarrow Q(x))$
- 18 a) Use mathematical induction to prove that $n^3 + 2n$ is a multiple of 3 for all $n \geq 1$. (5)
- b) Show that $(\exists x)(P(x) \wedge Q(x)) \implies (\exists x)P(x) \wedge (\exists x)Q(x)$ (5)
- 19 a) Test the consistency of the following statements. (5)
- “If Jack misses many shootings, then his films will be a flop. If Jack’s film flops, then he fails to act. If he signs for many films, then he does not fail to act.” Jack misses many shootings and signs for many films.
- b) Show that $R \rightarrow S$ can be derived from $P \rightarrow (Q \rightarrow S)$, $R \vee P$ and Q (5)
- 20 a) Using proof by contrapositive method, prove that “if $3n + 2$ is odd, then n is odd” (5)
- b) Use mathematical induction to prove that $1^2 + 2^2 + 3^2 + \dots + n^2 = (n/6)(n + 1)(2n + 1)$ for all $n \in \mathbb{N}$. (5)

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
THIRD SEMESTER B.TECH DEGREE EXAMINATION(S), MAY 2019

Course Code: CS201

Course Name: DISCRETE COMPUTATIONAL STRUCTURES

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks.

Marks

- | | | |
|---|---|-----|
| 1 | Show that set of all integers are countable. | (3) |
| 2 | What is the minimum number of students required in a class to be sure that at least six will receive same grade, if there are five possible grades. | (3) |
| 3 | With suitable example define GLB and LUB of a partially ordered set. | (3) |
| 4 | Show that $(E,+)$ E is set of all even non negative integers is subsemigroup of $(N,+)$ N is set of all natural numbers. | (3) |

PART B

Answer any two full questions, each carries 9 marks.

- | | | |
|---|--|-----|
| 5 | a) Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + (n+1)2^n$ | (5) |
| | b) Let $R = \{(1,2), (3,4), (2,2)\}$ $S = \{(4,2), (2,5), (3,1), (1,3)\}$ Find $RoS, SoR, Ro(SoR)$ and RoR | (4) |
| 6 | a) Let R be the set of real numbers and S is the relation on R defined by $(x,y) \in S$ if $(x-y)$ is divisible by 7. Prove that S is an equivalence relation. Find equivalent class of S. | (5) |
| | b) How many permutations are there for the eight letters a,e,f,g,i,t,w,x. How many start with letter 't' and how many start with letter 't' and end with letter 'c'. | (4) |
| 7 | a) Check whether the algebraic structure $\langle \mathbb{Z}_5, +_5 \rangle$ defined over the set of positive integers is a semigroup or not? | (5) |
| | b) Show that $A \times (B \cap C) = (A \times B) \cap (A \times C)$ | (4) |

PART C

Answer all questions, each carries 3 marks.

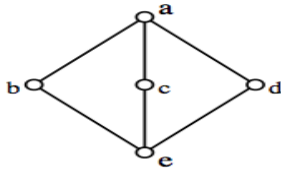
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|----|---|-----|
| 8 | Prove that inverse element of a group is unique | (3) |
| 9 | Define algebraic system with two binary operations. | (3) |
| 10 | Prove that every chain is a distributive lattice | (3) |
| 11 | Define a Boolean algebra. Give an example | (3) |

PART D

Answer any two full questions, each carries 9 marks.

- 12 a) State and prove Lagrange's Theorem (5)
 b) For any Boolean algebra B, prove that $a+b = a+c$ and $ab = ac \Rightarrow b = c$ for all $a, b, c \in B$ (4)

- 13 a) Check whether the following is distributive lattice or not (4)



- b) If $(a+b)^2 = a^2 + 2ab + b^2 \forall a, b, c \in R$, Prove that R is a commutative Ring and conversely (5)
 14 a) Prove that the set $G = \{0, 1, 2, 3, 4, 5\}$ is a abelian group under addition modulo 6. (5)
 b) Define lattice homomorphism. (4)

PART E

Answer any four full questions, each carries 10 marks.

- 15 a) Show that $n^3 + 2n$ is divisible by 3 by mathematical induction (5)
 b) Show that the following premises are inconsistent (5)
 1. If Jack misses many classes through illness, then he fails high school
 2. If Jack fails high school then he is uneducated
 3. If Jack reads a lot of books, then he is not educated
 4. Jack misses many class through illness and reads a lot of books
 16 a) Construct truth table for the following formula $(P \vee Q) \vee (\sim P \vee Q) \vee (P \wedge \sim Q)$ (5)
 b) Show that $R \rightarrow S$ can be derived from the premises $P \rightarrow (Q \rightarrow S)$, $\sim R \vee P$ and Q (5)
 17 a) Show that $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$ (5)
 b) Explain proof by contradiction with an example (5)
 18 a) Without using truth table prove that $(P \vee Q) \wedge \sim(\sim P \wedge (\sim Q \vee \sim R)) \vee (\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)$ is a tautology (5)
 b) Show that the conclusion C follows from the premises H1, H2
 H1: $\sim Q$ H2: $P \rightarrow Q$ C: $\sim P$ (5)
 19 a) Symbolize the expression "All the world loves a lover" (5)
 b) Prove that $5+10+15+\dots+5n = 5n(n+1)/2$ using mathematical induction (5)
 20 a) Determine the validity of the following arguments (5)
 Every living thing is a plant or animal
 John's gold fish is alive and it is not a plant
 All animals have hearts
 Therefore, John's gold fish has heart.
 b) Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q, Q \rightarrow R, P \rightarrow M$, and $\sim M$ (5)

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
THIRD SEMESTER B.TECH DEGREE EXAMINATION(R&S), DECEMBER 2019

Course Code: CS201

Course Name: DISCRETE COMPUTATIONAL STRUCTURES

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks.

Marks

- 1 Let $f(x) = x+2$ $g(x) = x-2$ and $h(x) = 3x$ for $x \in \mathbb{R}$, where \mathbb{R} is the set of real numbers. Find $f \circ h$, $h \circ g$, $f \circ (g \circ h)$ (3)
- 2 Let R be a relation on set A . Prove that if R is reflexive then R^{-1} is also reflexive (3)
- 3 In how many ways can 12 balloons be distributed at a birthday party among 10 children if we ensure that every child gets atleast one balloon. (3)
- 4 Show that the set \mathbb{N} of natural numbers is a monoid under the operation $x * y = \max(x, y)$. (3)

PART B

Answer any two full questions, each carries 9 marks.

- 5 a) Prove that (i) $A \oplus B$ or $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$ (5)
 (ii) $(A - C) \cup (B - C) = (A \cap B) - C$ Where A , B and C are any sets.
- b) How many permutations can be made with the letters of the word MISSISSIPPI taken all together? How many of these will be vowels occupying the even places? (4)
- 6 a) Let $X = \{ 2, 3, 6, 12, 24, 36 \}$ and the relation be such that $x \leq y$ if x divides y . Give the relation and draw the Hasse diagram of (X, \leq) . Also find the maximal and minimal elements. (4)
- b) Solve the recurrence relation $a_r + a_{r-1} = 3r \cdot 2^r$ using characteristic root method (5)
- 7 a) State Pigeon hole principle. Show that among any 11 numbers there exist at least 2 numbers with the same unit digit. (4)
- b) Let $A = \{ 1, 2, 3, \dots, 11, 12 \}$ and let R be the relation on $A \times A$ defined by $(a, b) R$ (5)

(c,d) iff $a+d = b+c$

(i) Prove that R is an equivalence relation

(ii) Find the equivalence class of (2,5)

PART C

Answer all questions, each carries 3 marks.

- 8 Show that the order of a subgroup of a finite group divides the order of the group. (3)
- 9 Prove that the set consisting of the fourth roots of unity forms an abelian group with respect to multiplication composition. (3)
- 10 In a distributive lattice $a \vee b = a \vee c$ and $a \wedge b = a \wedge c$ imply that $b = c$. (3)
- 11 Show that the complement of every element in a boolean algebra is unique. (3)

PART D

Answer any two full questions, each carries 9 marks.

- 12 a) The necessary and sufficient condition that a non-empty subset H of a Group G be a subgroup is $a \in H, b \in H \Rightarrow ab^{-1} \in H$ (5)
- b) Let x, y be arbitrary elements in a boolean algebra (B, +, ., ', 0, 1). Prove the De-Morgan's Law $(x+y)' = x'.y'$. (4)
- 13 a) Show that the lattice with three or fewer elements is a chain. (5)
- b) What is Ring with Unity? Give an example of a commutative ring without unity. (4)
- 14 a) If the order of a group G be 'n' ie $a^n=e$ then the set $H = \{ a, a^2, \dots, a^n \}$ forms a group with respect to the multiplication composition in G (5)
- b) Define a bounded lattice. Give an example. (4)

PART E

Answer any four full questions, each carries 10 marks.

- 15 a) Show the following implication without constructing the truth table (5)
- $$((P \vee \neg P) \rightarrow Q) \rightarrow ((P \vee \neg P) \rightarrow R) \Rightarrow (Q \rightarrow R)$$
- b) Negate the following statements and give the logical expression (5)

(i) All apples red. (ii) Some students are brilliant

16 a) Check the validity of the following argument: (5)

“If I get the job and work hard, then I will get promoted. If I get promoted, then I will be happy. Therefore, either I will not get the job or I will not work hard.”

b) Differentiate between free and bound variables with suitable examples. (5)

17 a) Derive the following using Rule CP if necessary (5)

(a) $\neg P \vee Q, \neg Q \vee R, R \rightarrow S \Rightarrow P \rightarrow S$ (b) $P \rightarrow (Q \rightarrow R), Q \rightarrow (R \rightarrow S) \Rightarrow P \rightarrow (Q \rightarrow S)$

b) Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$ using Indirect method of Proof. (5)

18 a) Show the following using indirect method of Proof (5)

$(R \rightarrow \neg Q), R \vee S, S \rightarrow \neg Q, P \rightarrow Q \Rightarrow \neg P$

b) Show by Mathematical Induction that $n^3 + n$ is divisible by 3. (5)

19 a) An island has two tribes of natives. Any native from the first tribe always tells the truth, while the other tribe always lie. If you arrive at the island and ask a native if there is gold on the island. He answers “ There is gold on the island if and only if I always tell the truth.” Is there gold on the island? Justify your answer with the help of truth table. (5)

b) show that from (i) $(\exists x)(F(x) \wedge S(x)) \rightarrow (y)(M(y) \rightarrow W(y))$ and (5)

(ii) $(\exists y)(M(y) \wedge \neg W(y))$ the conclusion $(x)(F(x) \rightarrow \neg S(x))$ follows.

20 a) Show that $\neg(P \wedge Q) \rightarrow \neg P \vee (\neg P \vee Q) \Leftrightarrow (\neg P \vee Q)$ (without constructing truth table) (5)

b) Show that “ If x is an integer and x^2 is even, then x is also even” by proof by contrapositive method. (5)

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Third semester B.Tech examinations (S) September 2020

Course Code: CS201**Course Name: DISCRETE COMPUTATIONAL STRUCTURES**

Max. Marks: 100

Duration: 3 Hours

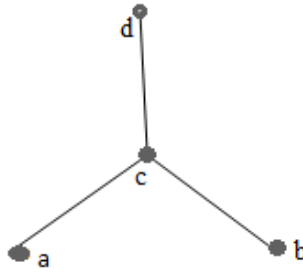
PART A*Answer all questions, each carries 3 marks.*

Marks

- 1 Show that $A \times (B \cap C) = (A \times B) \cap (A \times C)$ (3)
- 2 Prove that if a relation R on set A is transitive & irreflexive, then it is asymmetric. (3)
- 3 If 9 books are to be kept in 4 shelves, there must be atleast one shelf which contains atleast 3 books. Justify (3)
- 4 In how many ways can 6 boys and 4 girls sit in a row? In how many ways can they sit in a row if just the girls are to sit together? (3)

PART B*Answer any two full questions, each carries 9 marks.*

- 5 a) Consider f, g and h are functions on set of integers. (4)
 $f(n)=n+2$, $g(n)=n^2$, $h(n)=3n$. Determine fogoh, gofoh, hofog, fog.
- b) If $S=N \times N$ and the binary operation * is defined by $(a,b)*(c,d)=(ac,bd)$ for all a,b,c,d in N, show that $(S, *)$ is a semigroup. Is it monoid? (5)
- 6 a) Which of the following relations on $\{0,1,2,3\}$ are equivalence relation? Justify the answer. (4)
 $R1=\{(0,0),(1,1),(2,2),(3,3)\}$
 $R2=\{(0,0),(0,2),(2,0),(2,2),(2,3),(3,2),(3,3)\}$
 $R3=\{(0,0),(1,1),(1,2),(2,1),(2,2),(3,3)\}$
 $R4=\{(0,0),(1,1),(2,2),(3,3),(2,3),(3,2)\}$
- b) Find the number of integers between 1 and 250 both inclusive that are not divisible by any of the integers 2,3,5 (5)
- 7 a) Solve the recurrence relation $a_n - 2a_{n-1} = 3^n$; $a_1=5$ (5)
- b) Determine whether POSET represented by Hasse diagram given below have greatest element, least element, minimal element, maximal element. (4)



PART C

Answer all questions, each carries 3 marks.

- 8 Prove that the order of each sub group of a finite group G is a divisor of the order of group G . (3)
- 9 Let G be a group and suppose that a and b are any elements of G . Show if $(ab)^2 = a^2b^2$ then group is abelian. (3)
- 10 Let (L, \leq) be a lattice and a, b, c, d elements of L . (3)
Prove that if $a \leq c$ and $b \leq d$ then $a \vee b \leq c \vee d$
- 11 Define principle of duality in Boolean algebra. (3)

PART D

Answer any two full questions, each carries 9 marks.

- 12 a) Prove that the necessary and sufficient condition that a non-empty subset of a group G be a subgroup is $a, b \in H \Rightarrow ab^{-1} \in H$. (5)
- b) State and prove Absorption properties of lattice (4)
- 13 a) Is D_{12} a complemented lattice? Explain (4)
[D_{12} is set of divisors of 12 for the relation $R = \{(x, y) | x \text{ divides } y\}$]
- b) Show that $(Z, \bullet, *)$ is a ring where $a \bullet b = a + b - 1$ and $a * b = a + b - ab$ (5)
for every $a, b \in Z$ (Set of all integers).
- 14 a) Show that the set $\{1, 2, 3, 4, 5\}$ is not a group under addition modulo 6 (4)
- b) Define boolean algebra and explain how it is related to lattice. (5)

PART E

Answer any four full questions, each carries 10 marks.

- 15 a) Show that $(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)$ tautologically implies r without using truth table (5)
- b) Express the following statements in predicate logic. (5)
- i) Some students are clever.
- ii) All men are kind.
- iii) Some person in this class has visited the Grand Canyon.

- 16 a) Suppose x is a real number. Consider the statement “if $x^2=4$, then $x=2$ ” (5)
Construct the converse, inverse, contrapositive of the given statement
- b) Prove the following implication (5)
 $\forall x[P(x) \rightarrow Q(x)], \forall x[R(x) \rightarrow \neg Q(x)] \Rightarrow \forall x[R(x) \rightarrow \neg P(x)]$
- 17 a) Show that $p \rightarrow \neg s$ is a valid inference from premises $p \rightarrow (q \vee r)$, $q \rightarrow \neg p$, $s \rightarrow \neg r$ (5)
- b) Prove that $n(n+2)$ is divisible by 4 by mathematical induction, if n is any even positive integer. (5)
- 18 a) Show that $\neg p \wedge (\neg q \wedge r) \vee (q \wedge r) \vee (p \wedge r) \equiv r$ using truth table. (5)
- b) Given the premises (5)
P1: All men are selfish.
P2: All politicians are men.
Prove that the conclusion “All politicians are selfish” is a valid conclusion.
- 19 a) Show that $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$ is a tautology. (5)
- b) Prove that $\exists z Q(z)$ is a valid conclusion from the premises (5)
 $\forall x[P(x) \rightarrow Q(x)], \exists y P(y)$
- 20 a) If the product of two integers a and b is even, then either a is even or b is even. (5)
Prove the statement by contraposition.
- b) Using proof by contradiction method, prove that “if $3n+2$ is odd, then n is odd” (5)
