B

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION, DECEMBER 2015

Civil Engineering

10CE6103 Theory of Elasticity

Max. Marks: 60 Duration: 3 Hours

Part A (Modules I & II)

(Answer any two questions: 9 x 2 = 18 Marks)

1. a) Explain octahedral plane and octahedral stresses.

(3 marks)

b) The state of stress at a point referred to a xyz system is given as (all in units of MPa)

$\sigma_{xx} = 30$ $\sigma_{yy} = -10$ $\sigma_{zz} = 10$

$\tau_{xy} = 40$ $\tau_{yz} = 10$ $\tau_{zx} = -10$

Determine the stress tensor referred to a new x'y'z' coordinate system obtained by an anticlockwise rotation of xyz through an angle 30^0 about the y axis. (6 marks)

2. a) Derive the compatibility conditions for strain in 3D. (5 marks)

b) The strain tensor at a point in a body is given by

$$[\varepsilon_{ij}] = \begin{matrix} 0.0001 & 0.0002 & 0.0005 \\ 0.0002 & 0.0003 & 0.0004 \\ 0.0005 & 0.0004 & 0.0005 \end{matrix}$$

Determine the deviator and spherical strain tensors. (4 marks)

3. a) Explain 'stress invariants'. (3 marks)

b) Under what conditions are the following expressions for the components of strain at a point compatible?

$$\varepsilon_x = 2a_1xy^2 + a_2y^2 + 2a_3xy$$

$$\varepsilon_y = a_1x^2 + a_2x$$

$$\gamma_{xy} = a_4x^2y + a_5xy + a_1x^2 + a_6y$$

(6 marks)

Part B (Modules III & IV)

(Answer any two questions: 9 x 2 = 18 Marks)

4. a) State and explain Saint Venant's principle. (5 marks)

   b) Obtain the stiffness matrix and compliance matrix for isotropic materials (4 marks)

5. a) what is Airy's stress function? Explain its significance. (3 marks)

   b) A cantilever beam of uniform length *L*, depth *d* and thickness *b* carries a concentrated force *P* at free end. Determine the stress distribution in the beam by stress function approach. (6 marks)

6. a) Explain plane strain and plane stress problems. (4 marks)

   b) The stress tensor at a point is given as

$$[\sigma_{ij}] = \begin{bmatrix} 200 & 160 & -120 \\ 160 & -240 & 100 \\ -120 & 100 & 100 \end{bmatrix} \text{ kN/m}^2$$

   Determine the strain tensor at this point. Take $E=210\text{x}10^6$ kN/m^2 and $\mu=0.3$.

   (5 marks)

Part C (Modules V & VI)

(Answer any two questions: 12 x 2 = 24 Marks)

7. a) Explain axisymmetric problems in elasticity with examples. (6 marks)

   b) A thick cylinder of inner radius 10cm and outer radius 15cm is subjected to an internal pressure of 12MPa. Determine the radial and hoop stresses in the cylinder at the inner and outer surfaces. (6 marks)

8. a) Explain Prandtl's membrane analogy for torsional problems. (4 marks)

   b) Derive the expressions for shear stress and angle of twist per unit length for a uniform bar of equilateral triangular section subjected to a twisting moment *T*.

   (8 marks)

9. a) Derive the equations of equilibrium in polar co-ordinates. (6 marks)

   b) A shaft of elliptical cross section having semi major axis of 60cm and semi minor axis of 30cm is subjected to a torque of 1000Nm. Find the maximum shear stress developed in the shaft. (6 marks)

B

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION, DECEMBER 2016

CIVIL ENGINEERING

10CE6103: THEORY OF ELASTICITY

Max Marks : 60 Duration: 3 Hours

Part A

(Answer any two questions: 2 x 9 = 18 Marks)

1 a) Explain stress invariants **(3)**

b)At a point a given material, the three dimensional state of stress is given by $\sigma x=1$, $\sigma y= -3$, $\sigma z=4$, $\tau xy=2$, $\tau yz= -4$, and $\tau xz=1$ all in units of kPa. Find the principal stresses, principal plane and check for invariance. **(6)**

2 a) Explain the state of strain at a point **(3)**

b) A body is subjected to three-dimensional force and the state of stress at a point in it is represented as

$$\begin{bmatrix} 200 & 100 & 200 \\ 100 & -100 & 150 \\ 200 & 150 & -100 \end{bmatrix}$$

Determine the normal stresses, sharing stress and resultant stresses on the octahedral planes **(6)**

3 a) Derive compatibility conditions for strain in 3D **(4)**

b) The displacement field in micro units for a body is given by

$u= (2x^2+3y)i+(3+2z)j+(x^2+4y)k$. Determine the principal strain at(4,2,-1) and the direction of the minimum principal strain **(5)**

Part B

(Answer any two questions: 2 x 9 = 18 Marks)

4. a) State and explain Generalised Hooks law **(4)**

b) Analyse by the method of polynomials for Airy's stress function, a simply supported beam of length 2L and subjected to and of intensity "w" on top **(5)**

5 a) Given the stress function $\phi = (H/\pi) z \tan^{-1}(x/z)$.Determine whether stress function ϕ is admissible. If so determine the stresses **(5)**

b) Explain the significance of Airy's stress function **(4)**

6 a) Explain plane stress and plain strain problems **(6)**

b) Prove that $\phi = Cr^2$ is a legitimate Airy's stress function. Derive the stresses from it. **(3)**

Part C

(Answer any two questions : 2 x 12 = 24 Marks)

7 a) Derive the equations of equilibrium in polar co-ordinates. **(6)**

b) A Thick cylinder of internal diameter 180mm and external diameter 280 mm is subjected to an internal pressure of $7 N/mm^2$.Determine the variation of radial and hoop stresses in the cylinder wall **(6)**

8 a) Derive expressions for stress distribution in plates due to circular holes, with usual notations **(7)**

b) Explain Prandtl's membrane analogy for torsional problems **(5)**

9 a) A prismatic bar of elliptical cross section is subjected to a torque. Determine the shear stress distribution and warping of the cross section **(8)**

b)A shaft of elliptical cross section having semi major axis of 80cm and semi minor axis of 40cm is subjected to a torque of1500N/m. Find the maximum shear stress developed in the shaft **(4)**

B

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH. DEGREE EXAMINATION, DECEMBER 2017

CIVIL ENGINEERING

10CE6103: THEORY OF ELASTICITY

Max. Marks : 60 Duration: 3 Hrs.

Part A (Modules I - II)

*(Answer any **two** questions : 2 × 9 = 18 Marks)*

1. a) Derive the differential equations of equilibrium. (5 marks)

b) The state of stress at a point in a stressed body is given by the following stress components: σ_x =75 MPa, σ_y = 60 MPa, σ_z = 50 MPa, τ_{xy} = 25 MPa, τ_{yz} = −25 MPa and τ_{xz} = 30 MPa. Determine the normal and shear stresses on a plane having directions cosines of its outer normal as cos (N, x) =12/25, cos (N, y) = 15/25 and cos (N, z) =16/25. (4 marks)

2. a) Derive the compatibility conditions. (6 marks)

b) The strain components at a point are given by ε_x =0. 1, ε_y = − 0.05, ε_z = 0.05, γ_{xy} = 0.3, γ_{yz} =0.1, γ_{xz} = − 0.08. Determine the principal strains. (3 marks)

3. a) If the stress field is given by

$$\sigma_x = 3xy^2z + 2x, \qquad \tau_{xy} = 0$$
$$\sigma_y = 5xyz + 3y \qquad \tau_{yz} = \tau_{xz} = 3xy^2z + 2xy$$
$$\sigma_z = x^2y + y^2z$$

Determine whether these components of stress satisfy the equilibrium equations or not as the point (1, -1, 2). If not, determine the suitable body force required at this point so that these stress components are under equilibrium. (3 marks)

b) The state of stress at a point in a stressed body is given by the following stress components:

σ_x =20 MPa, σ_y = −40 MPa, σ_z = 80 MPa, τ_{xy} = 40 MPa, τ_{yz} = -60 MPa and τ_{xz} = 20 MPa. Determine the principal stresses, deviatoric and spherical stress tensors. (6 marks)

Part B (Modules III - IV)

*(Answer any **two** questions : 2 × 9 = 18 Marks)*

4. a) State and explain the St. Venant's principle. (3 marks)

b) The displacement at a point (x, y) are as given below

$$u = 5x^4 + 3x^2y^2 + x + y$$
$$v = y^3 + 2xy + 4$$

Compute the values of normal and shearing strains at a point (3, -2) and verify whether compatibility exists or not? (5 marks)

c) At a point in a stressed material, the state of strain is determined as follows:

$$\varepsilon_x = 0.001, \quad \varepsilon_y = -0.003, \quad \varepsilon_z = \gamma_{xy} = 0, \quad \gamma_{xz} = -0.004, \quad \gamma_{yz} = 0.001$$

Calculate the volumetric strain and the Lame's constants if $E = 210\ kN / mm^2$ and Poisson's ratio is 0.3. (2 marks)

5. a) State and explain the Generalised Hooke's law and deduce the simplifications possible for orthotropic, transversely isotropic and isotropic media. (5 marks)

b) Discuss a problem of plane stress that can be solved using a 4th degree polynomial. (4 marks)

6. Determine the stresses and displacements developed in a cantilever of span L, depth 2h and unit width subjected to a point load at the free end using Airy's stress function approach. (9 marks)

Part C (Modules V & VI)

(Answer any two questions : 2 × 12 = 24 Marks)

7. a) Is the following expression, a stress function?

$$\phi = -\left(\frac{P}{\pi}\right) r\theta \sin\theta$$

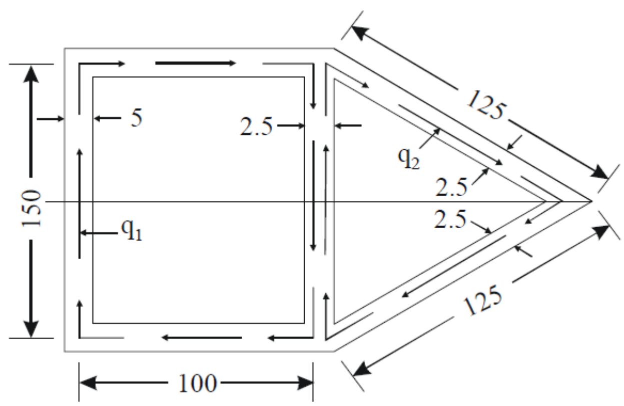
If so, find the corresponding stresses. (4 marks)

b) Derive the expression for stresses in a shaft of equilateral triangular cross-section under torsion and plot the variation of stress. (8 marks)

8. a) Derive the equations of equilibrium in polar coordinates. (8 marks)

b) The internal and external diameters of a thick hollow cylinder are 80 mm and 120 mm respectively. It is subjected to an external pressure of 40 MN/m^2, when the internal pressure is 120 MN/m^2. Calculate the circumferential stresses at the external and internal surfaces and determine the radial and circumferential stresses at the mean radius. (4 marks)

9. A two-cell tube as shown in the figure below is subjected to a torque of 10 kN-m. Determine the shear stress in each part and the angle of twist per metre length. Assume G = 83 kN/mm^2.



All dimensions in mm

(12 marks)

B

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION, DECEMBER 2017

CIVIL ENGINEERING

(Structural Engineering and Construction Management & Computer Aided Structural Engineering)

10CE6103 THEORY OF ELASTICITY

Max. Marks : 60 Duration: 3 Hours

Part A (Modules I - II)

(Answer any two questions : 9 x 2 = 18 Marks)

1) The state of stress at a point with respect to xyz coordinates is given by

$$\sigma = \begin{matrix} 30 & 20 & -20 \\ 20 & -15 & 10 \\ -20 & 10 & 10 \end{matrix} \text{ MPa}$$

a) Determine the stress tensor referred to a new x'y'z' co-ordinate system obtained by an anticlockwise rotation of xyz through an angle 30° about y axis.

b) Determine the hydrostatic and deviatoric stress components for the given state of stress.

(6+3 marks)

2a) Explain octahedral normal stresses and octahedral shear stresses.

b) Derive the strain displacement relations.

(4+5 marks)

3)a) Write the compatability relations which need to be satisfied by an admissible strain field.

b) The displacement components in an elastic body are given by

$u = (5xyz^2 + 3x^2yz) \times 10^{-4}$mm ; $v = (y^3z + 2x^2y) \times 10^{-4}$mm and $w = (4xyz + 2yz^2) \times 10^{-4}$mm where (x,y,z) is the coordinates of the point in mm. Evaluate the complete strain tensor at (1,2,3)mm. (4+5 marks)

Part B (Modules III - IV)

(Answer any two questions : 9 x 2 = 18 Marks)

4 a) Obtain the stiffness matrix and compliance matrix for isotropic materials

b) State and Explain Saint Venant's Principle. (5+4 marks)

5 a) Briefly describe plane strain problems.

b) Derive the governing equation for the compatibility condition to be satisfied by the Airy's stress function for plane strain problems. (4+5 marks)

6) Consider the cantilever beam with a moment M at the free end. Let b x d be the cross-section. Use the stress function $\Phi = Ay^3$ to arrive at the stress field. Determine the displacement field using appropriate boundary conditions.

(9 marks)

Part C (Modules V & VI)

(Answer any two questions : 12 x 2 = 24 Marks)

7) Determine the stress distribution around a small circular hole of radius 'a' in a plate of infinite dimensions when subjected to uniaxial tension S.

(12 marks)

8 a) Write the equilibrium equations, strain-stress relations and strain displacement equations in polar coordinates in 2D.

b) Obtain the angle of twist and maximum shear stress of a prismatic bar having a narrow rectangular cross-sectional area, when a torque T is applied.

(6+6 marks)

9 a) Explain Prandtl's membrane analogy for torsional problems

b) Determine the angle of twist per unit length and shear stress induced of a hollow shaft of uniform wall thickness 5mm. It is having a cross-section 100mm x 50mm and is subjected to a torque of 2kNm. The modulus of rigidity G = 1.3 x 10^4 MPa.

(7+5 marks)

B

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION, DECEMBER 2018

CIVIL ENGINEERING

(Structural Engineering and Construction Management & Computer Aided Structural Engineering)

10CE6103 THEORY OF ELASTICITY

Max. Marks : 60 Duration: 3 Hours

Part A (Modules I - II)

(Answer any two questions : 9 x 2 = 18 Marks)

1) The stress tensor at a point in a solid body with respect to the xyz co-ordinate is given by σ_x= 30; σ_y= 60; σ_z = -50; τ_{xy} = 15 ; τ_{yz} = 30; τ_{zx} = 10MPa. Find i) the stress invariants and prinicipal stresses ii) octahedral normal and shear stresses.

(9 marks)

2a) Write the relations which need to be satisfied by an admissible strain field.

b) Derive the stress transformation relation in 3D co-ordinates.

(4+5 marks)

3a) The displacement field for a body is given by

$u = \{(5x^2yz + 3xz^2)i + (4yz^3 - 2xz)j + (y^3z + 2xy)k\} \times 10^{-3}$ mm

Evaluate the components of strain tensor at the point P whose co-ordinates are (3,2,1).

b) Derive the traction boundary conditions

(5+4 marks)

Part B (Modules III - IV)

(Answer any two questions : 9 x 2 = 18 Marks)

4 a) Can we have two different solutions for elasticity problems. Explain.

b) Derive the biharmonic equation for two dimensional problems of elasticity in the absence of body forces.

(4+5 marks)

5 a) Explain the different types of boundary value problems of elasticity.

b) Derive the equations of equilibrium in terms of displacement.

(4+5 marks)

6) Differentiate between plane stress and plane strain problems. Write the stiffness and compliance matrix for both the problems.

(9 marks)

Part C (Modules V & VI)

(Answer any two questions : 12 x 2 = 24 Marks)

7) Determine the stress field for the problem of thick cylinder with inner radius 'a' and outer radius 'b' subjected to internal pressure p_i and external pressure p_o.

(12 marks)

8 a) Show that for a shaft of any arbitrary cross section, the torque transmitted is given by $T = 2\iint \Phi dxdy$ and the angle of twist by $\frac{-1}{2G}\{\Phi,xx + \Phi,yy\}$ where Φ is the stress function.

b) Obtain the angle of twist and maximum shear stress of a prismatic bar having an elliptical cross-section when subjected to a torque 20kNm. The semi major and semi minor axes of ellipse are 100mm and 50mm respectively. The modulus of rigidity $G = 1.3 \times 10^4$ MPa.

(6+6 marks)

9 a) Write the stress function and traction boundary conditions for the problem of large plate with a central hole of radius 'a' subjected to uniaxial tension 'S'.

b) How can we find solutions to torsion of hollow tubes with multiple holes by membrane analogy? Explain with necessary equations.

(5+7 marks)

B

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION, JULY 2019

CIVIL ENGINEERING

(**Computer Aided Structural Engineering)**

10CE6103 Theory of Elasticity

Max. Marks: 60 Duration:3 hours

Part A (Modules I - II)

(Answer any two questions : 2 x 9 = 18 Marks)

1. Using the elementary concept of strain, derive the strain displacement relations. (9 Marks)

2. The displacement field in a solid continuum is given by

$u=(x^2+y)\mathbf{i}+(3+z)\mathbf{j}+(x^2+2y)\mathbf{k}$.

Determine the principal strains at a point P (3, 1, -2) and the major principal direction.

(9 Marks)

3.a) Explain the term “stress transformation”. (6Marks)

b) Determine the stresses of the plane which is equally inclined to the principal planes.

(3 Marks)

Part B (Modules III - IV)

(Answer any two questions : 2 x 9 = 18 Marks)

4. a) Write down the stress strain relation for a linear isotropic element? (6 Marks)

b) Write a note on Saint Venants principle? (3 Marks)

5a) Derive the stress compatibility equation for plain stress problem with body force.

(5 marks)

b) Write a note on Airys stress function? (4 Marks)

6.a) Write down the stress strain relation through the fourth order elasticity tensor. Discuss the changes in the number of independent constants due to symmetry condition applicable for all linear elastic material. (6 Marks)

b) Write down the equation of Lames coefficients in terms of Youngs Modulus and Poissons ratio. (3 Marks)

Part C (Modules V & VI)

(Answer any two questions : 2 x 12 = 24 Marks)

7. a)Determine the stress , strain and the displacement field inside a thick cylinder(6 Marks)

b) Derive the equilibrium equation in polar coordinates (6 Marks)

8. a) Derive the expression for shear stress in prismatic triangular cross section. ? (8 Marks)

b) Derive the stress strain relations in polar co-ordinates? (4 Marks)

9. a) Explain Prandtls analogy. (8 Marks)

b)Explain the torsion in thin walled closed tubes.. (4 Marks)

M

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH. DEGREE EXAMINATION, DECEMBER 2019

COMPUTER AIDED STRUCTURAL ENGINEERING

10CE6103: THEORY OF ELASTICITY

(Assume any missing data)

Max. Marks: 60 Duration:3Hrs

Part A (Modules I - II)

(Answer any twoquestions: 2x9 = 18 Marks)

1. The stress tensor at a point in a solid body with respect to the XYZ co-ordinate is given by $\sigma_x = 30$, $\sigma_y = 60$, $\sigma_z = -50$, $\tau_{xy} = 15$, $\tau_{yz} = 30$, $\tau_{zx} = 10$ MPa. Find i) the stress invariants and principal stresses ii) octahedral normal and shear stresses. 9 Marks

2. a) The displacement field for a body is given by $u = \{(5x^2yz + 3xz^2)i + (4yz^3 - 2xz)j + (y^3z + 2xy)k\} \times 10^{-3}$ mm. Evaluate the components of strain tensor at the point P whose co-ordinates are (3,2,1). 4 Marks

   b) Explain state of stress at a point and Cauchy's stress tensor. 5 Marks

3. When the stress tensor at a point with reference to axes (x,y,z) is given by the array, $\begin{bmatrix} 4 & 1 & 2 \\ 1 & 6 & 0 \\ 2 & 0 & 8 \end{bmatrix}$ MPa. Show that the stress invariants remain unchanged by transformation of the axes by 45° about the z-axis. 9 Marks

Part B (Modules III - IV)

(Answer any twoquestions: 2x9 = 18 Marks)

4. a) Explain the three different types of boundary value problems of elasticity. 6 Marks

   b) Write the constitutive relationship for orthotropic and isotropic materials. 3 Marks

5. A large thin plate is subjected to certain boundary conditions on its thin edges (with its large faces free of stress), leading to the stress function $\phi = Ax^3y^2 - Bx^5$.
   i) Use the biharmonic equation to express A in terms of B
   ii) Calculate all stress components
   iii) Calculate all strain components
   iv) Check that the compatability equation is satisfied
   v) Check that the equilibrium equations are satisfied 9 Marks

6. a) Explain plane stress problem. Write any two examples of plane stress problems. 3 Marks

b) The displacement field in a homogeneous, isotropic, linearly elastic body is given by: u = $(3x^2z+60x)i + (5z^2+10xy)j + (6z^2 + 2xyz)k \times 10^{-6}$ mm. Evaluate the stress components σ_x, σ_y and τ_{xy} at the point P(-3, 10, -5) mm if the Lame's constants are $\lambda = 1.2 \times 10^5$ MPa and $\mu = 0.8 \times 10^5$ MPa. 6 Marks

Part C (Modules V & VI)

(Answer any two questions : 2 x12 = 24 Marks)

7. a) Derive the 2 dimensional differential equations of equilibrium in cylindrical polar coordinates. 7 Marks

   b) Explain Prandtl's membrane analogy? 5 Marks

8. a) A thick cylinder of inner radius 10 cm and outer radius 15 cm is subjected to an internal pressure of 12 MPa. Determine the radial and hoop stresses in the cylinder at the inner and outer surfaces. 8 Marks

   b) What are axisymmetric problems and write the two-dimensional strain-displacement relations in axisymmetric problems. 4 Marks

9. a) Derive the expression for the i) resultant shear stress distribution ii) the maximum shear stress subjected to a torque 'T' using Prandtl's stress function approach. 8 Marks

   b) What is warping deformation? Draw the contour lines of the warping function of a prismatic bar of elliptic cross-section. 4 Marks

Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

M.Tech S1 (R,S) Exam Dec 2020

Pages : 2 **B**

Reg. No……………… Name:……..…………………

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION

CIVIL ENGINEERING

10CE6103: THEORY OF ELASTICITY

Max. Marks : 60 Duration: 3 Hours

PART A (Modules I & II)

Answer any TWO full questions (2 x 9=18 Marks)

1 **(a)** Derive the Navier's equations of equilibrium **(4 Marks)**

(b) The components of strain tensor at appoint in a solid are given by **(5 Marks)**
$\epsilon_x = a(x^2+y^2)$, $\epsilon_y = b(y^2+z^2)$, $\epsilon_z = c(z^2+x^2)$, $\gamma_{xy} = dxy$, $\gamma_{yz} = eyz$, $\gamma_{zx} = fzx$.
What conditions need to hold between the constants a to f so that the above is a possible strain field.

2 **(a)** The state of stress at a point with respect to xyz coordinates is given by **(6 Marks)**

$$\sigma = \begin{matrix} 10 & -20 & 20 \\ -20 & 30 & 10 \\ 20 & 10 & 20 \end{matrix} \text{ MPa}$$

Determine the stress tensor referred to a new x'y'z' co-ordinate system obtained by an anticlockwise rotation of xyz through an angle 45^0 about y axis

(b) Determine the hydrostatic and deviatoric stress components for the given state of stress **(3 Marks)**

3 **(a)** Write the compatibility relations, which need to be satisfied by an admissible strain field. **(3 Marks)**

(b) The displacement components in an elastic body is described by: **(6 Marks)**
$u=5x^2z+3x^2z$; $v=5(x^2+z^2-xyz)$; $w = 3x^2(y+z)+2y^2(x+z)$;
Evaluate the complete strain tensor at (5,10,20) mm

PART B (Modules III & IV)

Answer any TWO full questions (2 x 9=18 Marks)

4 **(a)** Briefly describe plane stress problems. Write the equations of equilibrium, stress- strain relations and compatibility equations for this case **(6 Marks)**

	(b)	Explain about boundary value problems of elasticity	**(3 Marks)**
5	**(a)**	Derive the governing equation for the compatibility condition to be satisfied by the Airy's stress function for plane strain problems	**(4 Marks)**
	(b)	The displacement field in a homogeneous isotropic elastic body is given by u = [(3 x^2z+20xy)i +(5 z^2+10x y^2)j+(6 z^2+2xyz)k] x 10-6mm If E = 2 x 105 N/mm2, υ = 0.25, Evaluate the stress components at the point (5,0,10).	**(5 Marks)**
6	**(a)**	Determine the displacement field for a cantilever beam with dimensions b x d, loaded with concentrated load P at free end.	**(9 Marks)**

PART C (Modules V & VI)

Answer any TWO full questions (2 x 12=24 Marks)

7	**(a)**	Explain axisymmetric problems in elasticity with examples	**(6 Marks)**
	(b)	A thick cylinder of inner radius 10cm and outer radius 15cm is subjected to an internal pressure of 12MPa. Determine the radial and hoop stresses in the cylinder at the inner and outer surfaces	**(6 Marks)**
8	**(a)**	Explain Prandtl's membrane analogy for torsional problems.	**(6 Marks)**
	(b)	Write the traction boundary condition and displacement boundary conditions for the problem of pure bending of curved bars	**(6 Marks)**
9	**(a)**	The stress function for Lame's problem is obtained ϕ = A+B lnr+Cr^2+D r^2lnr. Obtain the stress field and displacement field for the problem. Give a simple argument based on the displacement field that the constant D must be zero for Lami's problem.	**(8 Marks)**
	(b)	Write the equations of equilibrium in polar-co-ordinates	**(4 Marks)**

Reg No.:_______________ Name:__________________________

Pages : 2 **APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY** **B**

M.Tech S1 (R,S) Exam Dec 2021

Reg. No……………… Name:……..…………………

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION

CIVIL ENGINEERING

10CE6103: THEORY OF ELASTICITY

Max. Marks : 60 Duration: 3 Hours

PART A (Modules I & II)

Answer any TWO full questions (2 x 9=18 Marks)

1 **(a)** Derive Cauchy's stress formula. **(5 Marks)**

(b) The stress at a point with reference to X = *(x, y, z)* is **(4 Marks)**

$$\begin{bmatrix} 0 & 20 & 30 \\ 20 & 50 & 0 \\ 30 & 0 & 60 \end{bmatrix}$$ MPa. Find the stress tensor for a set of coordinate axes X' = *(x', y', z')* rotated 30° about the *x*- axis anticlockwise.

2 The stress tensor at a point in a solid body with respect to the XYZ co-ordinate is given by σ_x = 40, σ_y = 70, σ_z = -30, τ_{xy} = 15, τ_{yz} = 35, τ_{zx} = 10 MPa. Find i) the stress invariants ii) principal stresses iii) octahedral normal stresses iv) octahedral shear stresses. **(9 Marks)**

3 **(a)** Name and state the conditions that are required to be satisfied by a strain field in order to ensure the existence of single valued continuous displacement components? **(4 Marks)**

(b) Show that hydrostatic stress is invariant under the transformation of the coordinate axes with the help of an example. **(3 Marks)**

(c) What are engineering shear strains and how are they different from tensorial shear strain? **(2 Marks)**

PART B (Modules III & IV)

Answer any TWO full questions (2 x 9=18 Marks)

4 **(a)** Derive Navier's equation for a three-dimensional problem of linear elasticity and briefly explain what it represents. **(9 Marks)**

5 **(a)** Explain plane stress and plane strain problems? Give examples with **(5 Marks)**

figures.

(b) Prove that the relation $\nabla^2 (\sigma_{xx} + \sigma_{yy}) = 0$ equation holds good for plane stress and plane strain with constant body force. **(4 Marks)**

6 (a) Show that a quadratic stress function leads to a constant stress field. **(4 Marks)**

(b) Explain Saint Venant's principle. **(2 Marks)**

(c) What are the three different boundary conditions that are used in elastostatics? **(3 Marks)**

PART C (Modules V & VI)

Answer any TWO full questions (2 x 12=24 Marks)

7 (a) Using Prandtl's stress function approach, derive the expression for the i) resultant shear stress distribution ii) the maximum shear stress subjected to a torque 'T'. **(6 Marks)**

(b) Derive the equations of equilibrium in polar coordinates for a two-dimensional problem. **(6 Marks)**

8 (a) Consider a circular shaft undergoing a torsion. The shaft is made of homogenous, isotropic and linearly elastic material. Let the cross section rotate about the longitudinal axis, the twist per unit length being α. A section at distance z from the fixed end will rotate through αz. Consider a typical cross section of this shaft and using Saint Venant's semi inverse method, describe the components of displacements in three directions. **(12 Marks)**

i) Define warping and what would be the warping in this case

ii) By utilizing warping function, state the displacement components

iii) Write down the corresponding strain components

iv) Using Hooke's law find out the stress components

v) Prove that warping function is harmonic

9 (a) Consider the following Airy stress function **(4 Marks)**

$$\emptyset = -\frac{P}{\pi} r\, \theta \sin\theta$$

Find the component of stress using the above stress function.

(b) A thick cylinder of inner radius 15 cm and outer radius 20 cm is subjected to an internal pressure of 25 MPa. Determine the radial and hoop stresses in the cylinder at the inner and outer surfaces. **(8 Marks)**
