

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
SIXTH SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

Course Code: CS302

Course Name: DESIGN AND ANALYSIS OF ALGORITHMS (CS)

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks.

- | | | Marks |
|---|---|-------|
| 1 | Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$? Justify your answer. | (3) |
| 2 | State Master's Theorem. Find the solution to the following recurrence equation using Master's theorem. | (3) |
| | a) $T(n) = 2T(n/2) + n \log n$ | |
| | b) $T(n) = 2^n T(n/2) + n^n$ | |
| 3 | Analyse the complexity of the following program | (3) |
| | <pre>main () { for (inti=1; i<=n;i=i*2) sum =sum+i+func(i) } void func(m) { for (int j=1; j<=m; j++) Statement with O(1) complexity } </pre> | |
| 4 | State weighted rule (union by rank) and collapsing rule (path compression) applied in the disjoint set union and find operation respectively. How these rules will improve the efficiency of disjoint set operations. | (3) |

PART B

Answer any two full questions, each carries 9 marks.

- | | | |
|---|---|-----|
| 5 | a) Using iteration solve the following recurrence equation | (5) |
| | $T(n) = 2$ if $n=1$ else $T(n) = 2T(n/2) + 2n + 3$ | |
| | b) Using Recursion Tree method, solve. | (4) |
| | Assume constant time for small values of n. | |
| | $T(n) = 2T(n/10) + T(9n/10) + n$ | |
| 6 | Construct a red-black tree by inserting the keys 41, 38, 31, 12, 19, 8 into an initially empty tree. Then show the red-black trees that result from the | (9) |

successive deletion of the keys in the order 8,12,1, 41.

- 7 a) Explain the important properties of B-Tree. (2)
 b) Construct a B-tree of minimum degree 3 by inserting the elements in the order given F, Q,P,K,A,L,R,M,N,X,Y,D,Z,E,H,T,V,W,C. from the constructed tree delete A,P,Q,R,T. (7)

PART C

Answer all questions, each carries 3 marks.

- 8 List and explain the characteristic properties associated with a problem that can be solved using dynamic programming. (3)
 9 Let G be a weighted undirected graph with distinct positive edge weights. If every edge weight is increased by same value, will the minimum cost spanning tree and shortest path between any pair of vertices change. Justify your answer. (3)
 10 Consider a complete undirected graph with vertex set $\{0, 1, 2, 3, 4\}$. Entry W_{ij} in the matrix W below is the weight of the edge $\{i, j\}$. What is the minimum possible weight of a spanning tree T in this graph such that vertex 0 is a leaf node in the tree T ? (3)

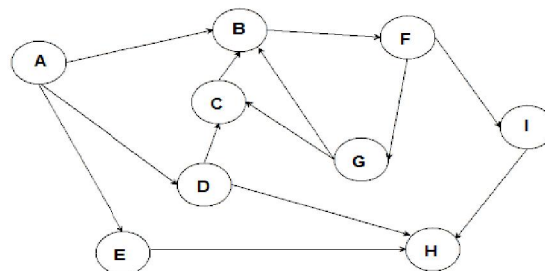
$$W = \begin{pmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{pmatrix}$$

- 11 Let (u,v) be a minimum-weight edge in a graph G . Show that (u,v) belongs to some minimum spanning tree of G . (3)

PART D

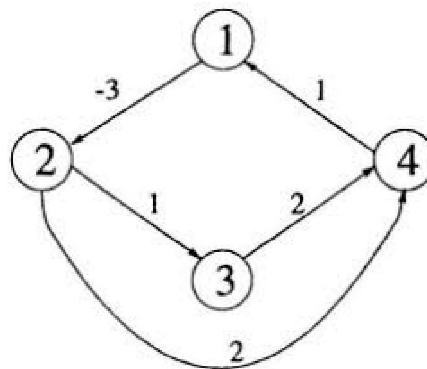
Answer any two full questions, each carries 9 marks.

- 12 a) Write down DFS algorithm and analyse the time complexity. What are different classification of edges that can be encountered during DFS operation and how it is classified? (4)
 b) (5)



Perform DFS traversal on the above graph starting from node A. Where multiple node choices may be available for next travel, choose the next node in alphabetical order. Classify the edges of the graph into different category.

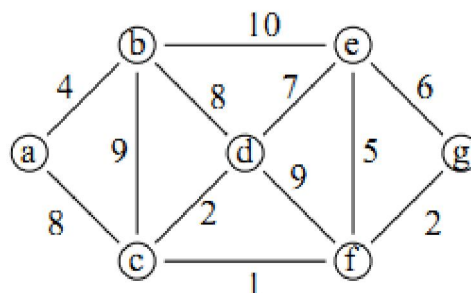
- 13 a) Write and explain an algorithm to find the optimal parenthesization of matrix chain product whose sequence of dimension is given. (5)
- b) Write and explain merge sort algorithm using divide and conquer strategy. Also analyse the complexity. (4)
- 14 a) Write down and explain Bellman Ford algorithm. Will your algorithm detect all negative cycles in the graph. Justify your answer. (5)
- b) Apply Bellman Ford algorithm on the graph given below. Assume Node 1 as source vertex. (4)



PART E

Answer any four full questions, each carries 10 marks.

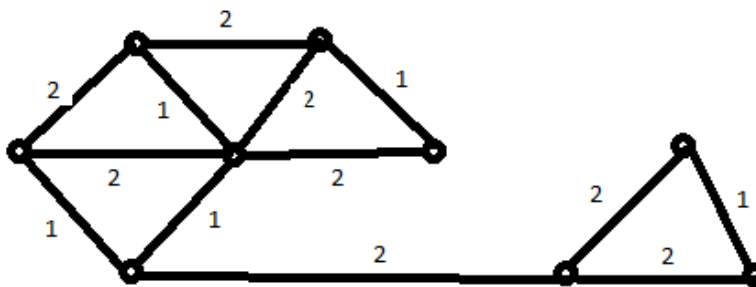
- 15 a) Write down Prim's algorithm and analyse the complexity. (4)
- b) Apply Prim's algorithm on the graph given below. (6)



- 16 a) Consider the following algorithm to determine whether or not an undirected graph has a clique of size k . First, generate all subsets of the vertices containing exactly k vertices. Next, check whether any of the sub-graphs induced by these subsets is complete (i.e. forms a clique). (4)
- Why is this not a polynomial-time algorithm for the clique problem, thereby

implying that $P = NP$?

- b) Prove that CLIQUE problem is NP-complete.
- 17 Explain the concept of Backtracking. Explain how 4 Queen problem can be solved using backtracking. Draw the state space tree corresponding to 4 Queen problem. (10)
- 18 Define Travelling Salesman Problem (TSP). Explain the basic steps that are to be followed to solve TSP using branch and bound. Illustrate with an example. (10)
- 19 a) State fractional knapsack problem. Give an algorithm for fractional knapsack problem using greedy strategy. (5)
- b) Find an optimal solution to the fractional knapsack problem for an instance with number of items 7, Capacity of the sack $W=15$, profit associated with the items $(p_1, p_2, \dots, p_7) = (10, 5, 15, 7, 6, 18, 3)$ and weight associated with each item $(w_1, w_2, \dots, w_7) = (2, 3, 5, 7, 1, 4, 1)$. (4)
- 20 a) Find the number of distinct minimum spanning trees for the weighted graph below (4)



- b) Consider a weighted complete graph G on the vertex set $\{v_1, v_2, \dots, v_n\}$ such that the weight of the edge (v_i, v_j) is $2|i-j|$. Find the weight of a minimum spanning tree of G . (3)
- c) Specify the difference between divide and conquer strategy and dynamic programming. (3)

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
SIXTH SEMESTER B.TECH DEGREE EXAMINATION(R&S), MAY 2019

Course Code: CS302

Course Name: DESIGN AND ANALYSIS OF ALGORITHMS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks.

- | | | Marks |
|---|---|-------|
| 1 | Define the terms Best case, Worst case and Average case time complexities. | (3) |
| 2 | What is the smallest value of n such that an algorithm whose running times is $100n^2$ runs faster than an algorithm whose running time is 2^n on the same machine? | (3) |
| 3 | State Master Theorem. | (3) |
| 4 | Explain the UNION and FIND-SET operations in the linked-list representation of disjoint sets. Discuss the complexity. | (3) |

PART B

Answer any two full questions, each carries 9 marks.

- 5 a) Determine the time complexities of the following two functions fun1() and fun2(): (2)

```
int fun1(int n)
{
    if (n <= 1) return n;
    return 2*fun1(n-1);
}
```

```
int fun2(int n)
{
    if (n <= 1) return n;
    return fun2(n-1) + fun2(n-1);
}
```

- b) Find the solution to the recurrence equation using iteration method: (3)
 $T(2^k) = 3 T(2^{k-1}) + 1,$
 $T(1) = 1$
- c) Solve the recurrence using recursion tree method: (4)
 $T(1) = 1$
 $T(n) = 3T(n/4) + cn^2$
- 6 a) Determine the best case and worst-case time complexity of the following function: (3)

```
void fun(int n, int arr[])
{
    int i = 0, j = 0;
    for(; i < n; ++i)
        while(j < n && arr[i] < arr[j])
            j++;
}
```

- }
 b) Explain the advantages of using height Balanced Trees? Explain AVL Rotations. (4)
 c) Find the minimum and maximum height of any AVL-tree with 7 nodes? Assume that the height of a tree with a single node is 0. (2)
- 7 a) List the Properties of B-Trees. (2)
 b) A 2-3-4 tree is defined as a B-Tree with minimum degree $t=2$. Create a 2-3-4 tree by successively inserting the elements (in the given order) 42, 56, 24, 89, 1, 5, 87, 8, 61, 6, 78, 7, 12, 34. (4)
 c) Delete the elements 89, 78, 12 and 8 from the above resultant tree. (3)

PART C

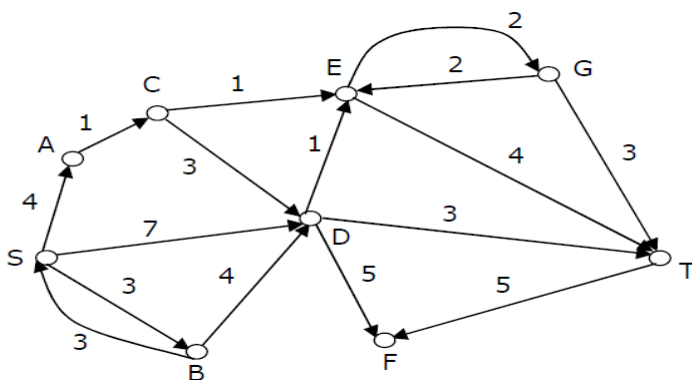
Answer all questions, each carries 3 marks.

- 8 In a weighted graph, assume that the shortest path from a source 's' to a destination 't' is correctly calculated using a shortest path algorithm. Is the following statement true? If we increase weight of every edge by 1, the shortest path always remains same. Justify your answer with proper example. (3)
- 9 Define Strongly Connected Components of a graph. (3)
 Write the algorithm to find Strongly Connected Components in a graph.
- 10 Write an algorithm to merge two sorted arrays and analyse the complexity. (3)
- 11 Write notes on Dynamic Programming Approach. List the sequence of steps to be followed in Dynamic Programming. (3)

PART D

Answer any two full questions, each carries 9 marks.

- 12 a) State Shortest Path Problem and Optimal substructure of Shortest Path. (2)
 b) Write Dijkstra's Single Source Shortest path algorithm. Analyse the complexity. (4)
 c) Find the shortest path from s to all other vertices in the following graph using Dijkstra's Algorithm. (3)



- 13 a) Write the algorithm for DFS and analyse its complexity. (4)
 b) Multiply the following two matrices using Strassen's Matrix Multiplication Algorithm. (5)

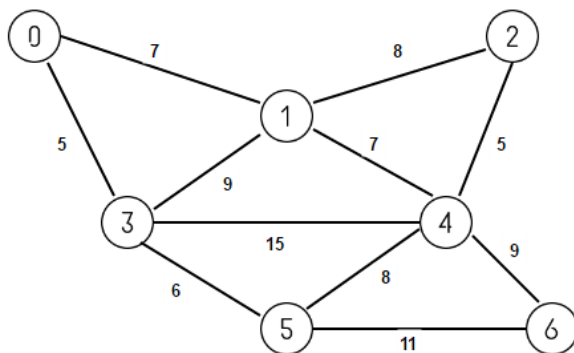
$$A = \begin{bmatrix} 6 & 8 \\ 9 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 5 \\ 3 & 6 \end{bmatrix}$$

- 14 a) State Matrix Chain Multiplication Problem. Write Dynamic Programming Algorithm for Matrix Chain Multiplication Problem. (4)
- b) Using Dynamic Programming, find the fully parenthesized matrix product for multiplying the chain of matrices $\langle A_1 A_2 A_3 A_4 A_5 A_6 \rangle$ whose dimensions are $\langle 30 \times 35 \rangle$, $\langle 35 \times 15 \rangle$, $\langle 15 \times 5 \rangle$, $\langle 5 \times 10 \rangle$, $\langle 10 \times 20 \rangle$ and $\langle 20 \times 25 \rangle$ respectively. (5)

PART E

Answer any four full questions, each carries 10 marks.

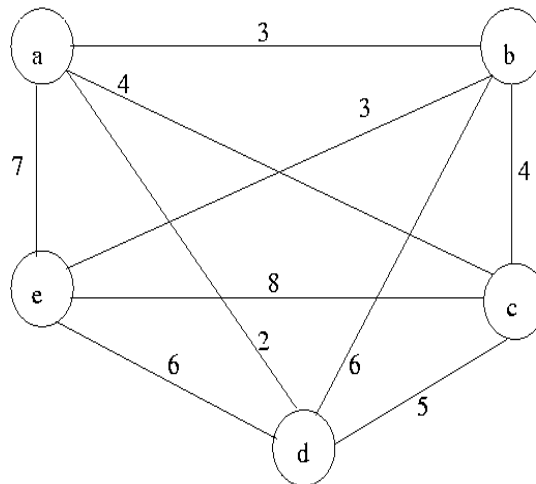
- 15 a) Explain Greedy Approach. Write the general greedy algorithm. (3)
- b) Formulate Fractional Knapsack Problem. Write Greedy Algorithm for fractional Knapsack Problem. (4)
- c) Find the optimal solution for the following fractional Knapsack problem. (3)
 $n=4$, $m = 60$, $W = \{40, 10, 20, 24\}$ and $P = \{280, 100, 120, 120\}$
- 16 a) Write the Kruskal's algorithm for Minimum Spanning Tree. Analyse its complexity. (6)
- b) Compute the Minimum Spanning Tree and its cost for the following graph using Kruskal's Algorithm. Indicate each step clearly. (4)



- 17 a) An undirected graph $G=(V, E)$ contains n ($n > 2$) nodes named v_1, v_2, \dots, v_n . Two vertices v_i, v_j are connected if and only if $0 < |i - j| \leq 2$. Each edge (v_i, v_j) is assigned a weight $i + j$. What will be the cost of the minimum spanning tree (as a function of n) of such a graph with n nodes? (4)
- b) Consider a complete undirected graph with vertex set $\{0, 1, 2, 3, 4\}$. Entry w_{ij} in the matrix W below is the weight of the edge $\{i, j\}$. What is the Cost of the Minimum Spanning Tree T using Prim's Algorithm in this graph such that vertex 0 is a leaf node in the tree T ? (6)

$$W = \begin{pmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{pmatrix}$$

- 18 a) State and Explain N Queens Problem. Write the backtracking algorithm for solving N Queens problem. (5)
- b) Show the state space tree for 4 Queens problem. Show the steps in solving 4 Queens problem using backtracking method to print all the solutions. (5)
- 19 a) Explain Branch and Bound method for solving Travelling Salesman Problem. (5)
- b) Solve Travelling Salesman problem for the following graph using Branch and Bound Technique. (5)



- 20 a) Define NP- Hard and NP – Complete Problems. (2)
- b) What are the steps used to show a given problem is NP-Complete? (4)
- c) Write notes on polynomial time reducibility. Give Examples. (4)

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
SIXTH SEMESTER B.TECH DEGREE(S) EXAMINATION(S), DECEMBER 2019

Course Code: CS302

Course Name: DESIGN AND ANALYSIS OF ALGORITHMS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks.

Marks

- | | | |
|---|---|-----|
| 1 | Analyse the complexity of the following function
<pre>void function(int n) { int count = 0; for (int i=n/2; i<=n; i++) for (int j=1; j<=n; j = 2 * j) for (int k=1; k<=n; k = k * 2) count++; }</pre> | (3) |
| 2 | Solve using Iteration method $T(n)=2T(n/2)+n, T(1)=1$ | (3) |
| 3 | Define B-tree. Discuss the significance of B-tree | (3) |
| 4 | Explain Asymptotic notations in algorithm analysis | (3) |

PART B

Answer any two full questions, each carries 9 marks.

- | | | |
|---|---|--|
| 5 | <p>a) Solve using Recursion Tree method (5)</p> $T(n)=3T(n/4)+n^2$ | |
| | <p>b) Analyse the complexity of the following functions (4)</p> <p>i) function(int n)</p> <pre>{ if (n==1) return; for (int i=1; i<=n; i++) { for (int j=1; j<=n; j++) { printf("*"); break; } } }</pre> <p>ii) void function(int n)</p> <pre>{ int i = 1, s = 1; while (s <= n) { i++; s += i; printf("*"); } }</pre> | |

- 6 a) Construct a Red Black tree by inserting 10,20,30,15,16 and 27 into an initially empty tree and also delete 15,16 and 30 from the tree (9)
- 7 a) Solve using Masters theorem (5)
- $T(n)=2T(n/4)+\sqrt{n}$
 - $T(n)=7T(n/2)+n^2$
- b) Explain AVL rotations with examples (4)

PART C

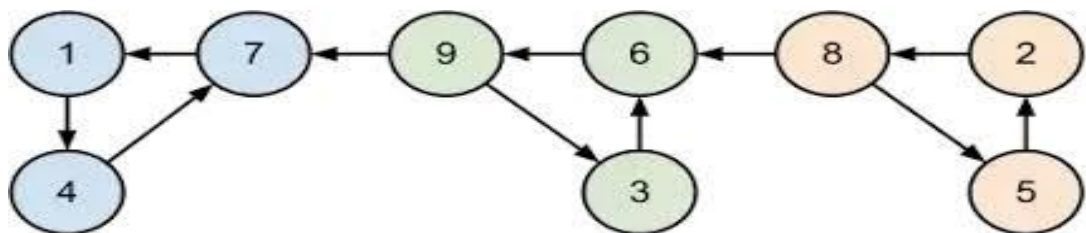
Answer all questions, each carries 3 marks.

- 8 Define spanning tree of a graph. Write the total number of spanning trees possible for a complete graph with 6 vertices. (3)
- 9 Write the applications of BFS and DFS (3)
- 10 List and explain the characteristic properties associated with a problem that can be solved using dynamic programming. (3)
- 11 Explain Divide and Conquer strategy. (3)

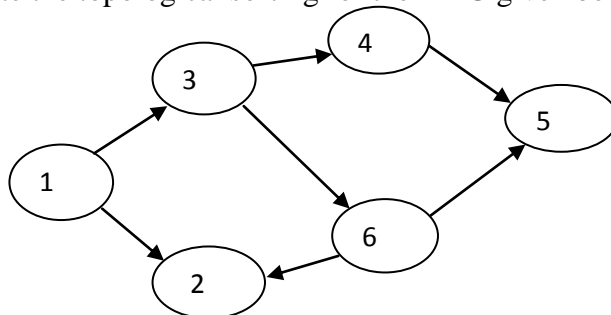
PART D

Answer any two full questions, each carries 9 marks.

- 12 a) What are different classification of edges that can be encountered during DFS operation and how it is classified? Explain with example (4)
- b) Find strongly connected components of the digraph using the algorithm showing each step (3)



- c) Write the topological sorting for the DAG given below (2)

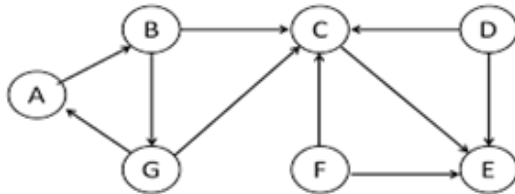


- 13 a) Given a chain of 4 matrices $\langle A_1, A_2, A_3, A_4 \rangle$ with dimensions $\langle 5 \times 4 \rangle, \langle 4 \times 6 \rangle, \langle 6 \times 2 \rangle, \langle 2 \times 7 \rangle$ respectively. Using Dynamic programming find the minimum number of scalar multiplications needed and also write the optimal multiplication order. (5)

- b) Write down Bellman Ford algorithm and analyse the complexity. What is the time complexity of Bellman-Ford single-source shortest path algorithm on a complete graph of n vertices? (4)

- 14 a) Write a short note on graph traversals (2)

Perform BFS traversal on the above graph starting from node A. If multiple node choices may be available for next travel, choose the next node in alphabetical order. (2)



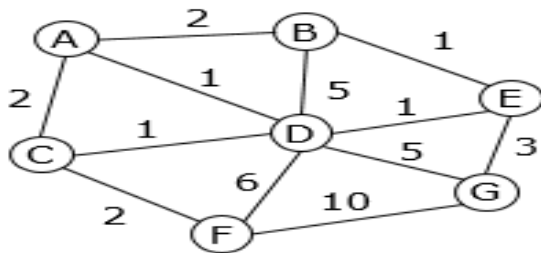
- b) Explain Strassen's matrix multiplication and analyse its complexity (5)

PART E

Answer any four full questions, each carries 10 marks.

- 15 a) Give a comparison between dynamic programming and Divide and conquer strategy (4)

- b) Apply Prim's algorithm on the following graph. Let A be the source vertex (6)



- 16 a) Formulate Fractional Knapsack Problem. Write Greedy Algorithm for fractional Knapsack Problem. (5)

- b) Find the optimal solution for the following fractional Knapsack problem. Given number of items $(n)=4$, capacity of sack $(m) = 60$, $W=\{40,10,20,24\}$ and $P=\{280,100,120,120\}$ (5)

- 17 a) Define NP hard and NP-Complete problems (4)

- b) Write short notes on Polynomial time reductions with example (4)

- c) Define class P and class NP (2)

- 18 a) Define Travelling Salesman Problem (TSP). Explain the basic steps that are to be followed to solve TSP using branch and bound with an example. (10)
- 19 a) Write the Kruskal's algorithm for finding minimum cost spanning tree and explain the Kruskal's algorithm with an example. Analyse the complexity of the algorithm. (7)
- b) Explain greedy strategy (3)
- 20 a) State and explain N-queens problem (2)
- b) Draw the state space tree of 4-queens problem (5)
- c) Define backtracking strategy (3)

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Sixth semester B.Tech examinations (S), September 2020

Course Code: CS302**Course Name: DESIGN AND ANALYSIS OF ALGORITHMS**

Max. Marks: 100

Duration: 3 Hours

PART A*Answer all questions, each carries 3 marks.*

Marks

- 1 Express the return value of the function “mystery” in θ – notation. (3)
- ```

intmystery(int n) {
int j=0,total=0;
for (int i=1;j<=n;i++){
++total;
j+=2*i;
}
return total;
}

```
- 2 Is  $2^{n+1} = O(2^n)$ ? Is  $2^{2n} = O(2^n)$ ? Justify (3)
- 3 Define a B-tree. Give an example. (3)
- 4 Implement UNION using linked list representation of disjoint sets. (3)

**PART B***Answer any two full questions, each carries 9 marks.*

- 5 a) Solve  $T(n) = 2T(n/2) + 2$  if  $n > 2$  (4)  
 $= 1$  if  $n = 2$  Using iteration method.
- b) Solve  $T(n) = 2T(\sqrt{n}) + \log n$  (5)
- 6 a) Show the red-black tree that result after successively inserting the keys (9)  
41,38,31,12,19,8 into an initially empty red-black tree.
- 7 a) Consider the following C function (4)
- ```

intcheck(int n){
inti,j;
for (i=1;i<=n;i++){

```

```

    for (j=1;j<n;j+=i){
printf("%d",i+j);
    }
}
}

```

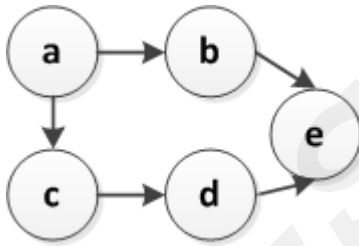
Find the time complexity of **check** in terms of θ – notation.

- b) Find the minimum and maximum height of any AVL-tree with 11 nodes. (5)
Assume that height of the root is 0.

PART C

Answer all questions, each carries 3 marks.

- 8 What is principle of optimality? (3)
9 Explain the characteristics of problems that can be solved using dynamic programming. (3)
10 Find the possible topological orderings for the following graph (3)



- 11 How the edges of a graph can be classified based on DFS? (3)

PART D

Answer any two full questions, each carries 9 marks.

- 12 a) Give a control abstraction for Divide and Conquer method. Explain with an example. (5)
b) Explain the effect of negative weight edges and negative weight cycles on shortest paths. (4)
13 a) Define strongly connected components. How DFS can be used to find strongly connected components? (4)
b) Find an optimal paranthesization of a matrix-chain product whose sequence of dimensions is $4 \times 10, 10 \times 3, 3 \times 12, 12 \times 20, 20 \times 7$. (5)
14 a) Write Dijkstra's Single Source Shortest path algorithm. Analyse the complexity. (7)

- b) Is it possible to find all pairs of shortest paths using Dijkstra's algorithm? (2)
Justify.

PART E

Answer any four full questions, each carries 10 marks.

- 15 a) Compare Divide and Conquer and Dynamic programming methodologies. (4)
b) Write an algorithm to merge 2 sorted arrays into a single sorted array. (6)
- 16 a) Explain Branch and bound technique. (3)
b) How Travelling Salesperson Problem can be solved using Branch and bound. (7)
- 17 a) Explain Kruskal's algorithm with an example. (6)
b) Derive its complexity of kruskal's algorithm.. (4)
- 18 a) Explain control abstraction of greedy method. (3)
b) Write greedy algorithm for knapsack problem. (4)
c) Find an optimal solution to the knapsack instance $n=7$, (3)
 $m=15, (p_1, p_2, \dots, p_7) = (10, 5, 15, 7, 6, 18, 3)$ and $(w_1, w_2, \dots, w_7) = (2, 3, 5, 7, 1, 4, 1)$.
- 19 a) Explain the concept of backtracking. (3)
b) How backtracking can be used to solve N-queens problem. (4)
c) Draw the state space tree for 4 Queens problem. (3)
- 20 a) Define NP-Hard and NP-complete problems. (4)
b) With examples explain polynomial time reducibility. (4)
c) What do you mean by intractable problems? (2)
