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Reg. No. : ..... **Name :** .....

SECOND SEMESTER B.TECH. DEGREE EXAMINATION, MAY/JUNE 2016

MA 102 : DIFFERENTIAL EQUATIONS

Max. Marks : 100 Duration : 3 Hours

PART – A

Answer **all** questions and **each** question carries **3** marks.

1. Determine a linearly independent solution of the differential equation $(x^2 + 1)\, y^{II} - 2xy^{I} + 2y = 0$ if $y_1 = x$ is solution.
2. Solve the differential equation $y^{IV} + 6y^{III} + 9y^{II} = 0$.
3. Find the particular integral of the differential equation $(D^2 - 2D + 1)y = xe^x$.
4. Solve by the method of variation parameters, $(D^2 + 4)y = \tan 2x$.
5. Develop the Fourier series of $f(x) = x^2$ in $-2 \le x \le 2$.
6. Find the Fourier sine series of $f(x) = e^x$ in $0 < x < 1$.
7. Obtain the partial differential equation by eliminating f and g from $z = xf(y) + yg(x)$.
8. Solve the partial differential equation $(y^2 + z^2)p - xyq + xz = 0$.
9. Obtain the solution of the wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ using method of separation of variables when the separation constant $k < 0$.
10. Write any two assumptions involved in deriving one dimensional wave equation.
11. Find the steady state temperature distribution in a rod of length 20 cm if the ends of the rod are kept at 10° C and 70° C.
12. Solve $\frac{\partial u}{\partial t} = h\frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(0, t) = u(1, t) = 0$ for $t > 0$ and $u(x, 0) = 3 \sin n\pi x$, $0 < x < 1$. **(12x3 = 36 Marks)**

P.T.O.

PART – B

Answer **six** questions – **one full** question from **each** Module.

Module – 1

13. a) Reduce to first order and hence solve the ODE
    i) $y^{II} + (y^{I})^3 \cos y = 0$ and
    ii) $2xy^{II} = 3y^{I}$.

    b) Solve the IVP $y^{II} - 2y^{I} + 5y = 0$, $y(0) = -3$, $y^{I}(0) = 1$. **11**

OR

14. a) Show that the functions x and x ln (x) are linearly independent (use Wronskian). Hence form an ODE for the given basis x, x ln (x).

    b) Solve the IV $Py^{II} + 0.2\, y^{I} + 4.01\, y = 0$, $y(0) = 0$, $y^{I}(0) = 2$. **11**

Module – 2

15. a) Solve the differential equation $(D + 1)^2 y = x^2 e^x$.

    b) Solve the differential equation $(x^3D^3 + 3x^2D^2 + xD + 1)y = x + \log x$. **11**

OR

16. a) Solve the differential equation $(D^2 + 1)y = x^2 e^x + \sin x$.

    b) Solve the differential equation $(x + 1)^2 y^{II} + (x + 1)y^{I} - y = 2 \sin \log (x + 1)$. **11**

Module – 3

17. a) Find the Fourier Series of $f(x) = \begin{cases} x & , 0 < x < 1 \\ 1 - x & , 1 < x < 2 \end{cases}$.

    b) Find the Fourier cosine series of $f(x) = x(\pi - x)$ in $0 < x < \pi$. **11**

OR

18. a) Expand $f(x) = e^{-x}$ in $(-l, l)$ as a Fourier Series.

    b) Find the half range sine series of $f(x) = x \sin x$ in $0 < x < \pi$. **11**

Module – 4

19. a) Form the PDE by eliminating a, b, c from $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

b) Solve the partial differential equation $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x \partial y}-2\frac{\partial^2 z}{\partial y^2}=e^{2x+y}$. 11

OR

20. a) Solve : $x(y^2-z^2)p + y(z^2-x^2)q = z(x^2-y^2)$.

b) Solve the partial differential equation $\frac{\partial^3 z}{\partial x^3}-4\frac{\partial^3 z}{\partial^2 x \partial y}+4\frac{\partial^3 z}{\partial x \partial y^2}=\cos(2x+y)$. 11

Module – 5

21. A tightly stretched string of length 'a' with fixed ends is initially in equilibrium position. Find the displacement u(x, t) of the string if it is set vibrating by giving each of its points a velocity $v_0 \sin(\pi x/a)$. 10

OR

22. A transversely vibrating string of length 'a' is stretched between two points A and B. The initial displacement of each point of the string is zero and the initial velocity at a distance x from A is kx(a – x). Find the form of the string at any subsequent time. 10

Module – 6

23. Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature zero if the initial temperature is $f(x)=\begin{cases} x & , 0<x<L/2 \\ L-x & , L/2<x<L \end{cases}$ 10

OR

24. An insulated rod of length L has its ends A and B maintained at 0° C and 100° C respectively until steady state conditions prevails. If B is suddenly reduced to 0° C and maintained at 0° C, then find the temperature in the rod at a distance x from A at time t. 10

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Reg. No._____ Name: _____

SECOND SEMESTER B.TECH DEGREE EXAMINATION, JULY 2016
Course Code: MA-102
Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100 Duration: 3 hrs

PART A
Answer all questions Each carries 3 marks

(1) Find the general solution of $y''' - y = 0$

(2) Find the wronskian of the following $e^{-x}\cos 5x$; $e^{-x} sin5\ x$

(3) Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x}$

(4) Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = x^2$

(5) Express $f(x) = x$ as aFourier series in the interval $-\pi < x < \pi$

(6) Obtain the half range Fourier sine series for the function e^x in $0 < x < 2$

(7) Form the partial differential equation by eliminating the arbitrary function from $z = y^2 + 2f\left(\frac{1}{x} + logy\right)$

(8) Solve $p\sqrt{x} + q\sqrt{y} = \sqrt{z}$

(9) Using the method of separation of variables solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x, 0) = 3e^{-5x}$

(10) State the one dimensional wave equation with boundary conditions and initial conditions for solving it

(11) In the Heat equation $\frac{\partial u}{\partial t} = \propto^2 \frac{\partial^2 u}{\partial x^2}$ what does $\propto^2$ indicate. State the boundary and initial conditions for solving it

(12) Find the steady state temperature distribution in a rod of length 25cm, if the ends of the rod are kept at 20^0c and 70^0c.

PART B
Answer one full question from each module

Module -I

(13) (a) Solve $y''' - 8y'' + 37\ y' - 50\ y = 0$ (6)

(b)Determine all possible solutions to the initial value problem $y' = 1 + y^2, y(0) = 0 \ \ in \ \ |x| < 3, |y| < 2$ (5)

OR

(14) (a)Find the general solution of $y^{iv} - y''' - 9\ y'' - 11\ y' - 4\ y = 0$ (6)

(b)Determine all possible solutions to the initial value problem

$\boldsymbol{y' = y^{\frac{1}{2}}, y(0) = 0.}$ (5)

Module - II

(15) (a)Solve by method of variation of parameters $\frac{d^2y}{dx^2} + y = xsinx.$ (6)

(b)Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x.$ (5)

OR

(16) (a)Solve $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2\log x.$ (6)

(b)Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = sin3xsin2x$. (5)

Module - III

(17) (a)Obtain the Fourier series for the function f(x) given by

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi} & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi} & 0 \le x \le \pi \end{cases}$$ (6)

(b)Obtain the Fourier series to represent the function

$$f(x) = |sinx|; -\pi < x < \pi$$ (5)

OR

(18) (a)Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $-\pi \le x \le$ (6)

(b)Find the half range cosine series for the function $f(x) = x^2$ in the range $0 \le x \le \pi$ (5)

Module - IV

(19) (a)Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 5e^{3x} - 7x^2 y.$ (6)

(b)Solve $(x+y)zp + (x-y)zq = x^2 + y^2$ (5)

OR

(20) (a) Solve $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 2\sin(3x + 2y).$ (6)

(b)Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = cos2x\ cos3y.$ (5)

Module – V

(21) A tightly stretched string with fixed end points $\boldsymbol{x = 0\ and\ x = l}$ is initially in a position given by $y = y_0 sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement $\boldsymbol{y(x,t)}$. (10)

OR

(22) A tightly stretched string with fixed end points $\boldsymbol{x = 0\ and\ x = l}$ *is* initially at rest in its equilibrium position. If it is vibrating by giving to each of its points a velocity $\boldsymbol{\lambda x(l-x)}$, find the displacement of the string at any distance x from one end at any time $\boldsymbol{t}$. (10)

Module - VI

(23) A bar 10 cm long with insulated sides has its ends A and B maintained at 30^0c and 100^0 c respectively until steady state conditions prevail. The temperature at A is suddenly raised to 20^0 c and at the same time that of B is lowered to 40^0 C. Find the temperature distribution in the bar at time t. (10)

OR

(24) A rod of 30cm long has its ends A and B kept at 30^0 c and 90^0 c respectively until steady state temperature prevails. The temperature at each end is then suddenly reduced to zero temperature and kept so. Find the resulting temperature function $\boldsymbol{u(x,y)}$ taking $x = 0$ at A. (10)

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Reg. No._____ Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SECOND SEMESTER B.TECH DEGREE SPECIAL EXAMINATION, AUGUST 2016

Course Code: MA-102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100 Duration: 3 hrs

PART A

Answer all questions Each carries 3 marks

1. Find ordinary differential equation for the basis $e^{-x\sqrt{2}},\ xe^{-x\sqrt{2}}$
2. Reduce $y'' = y'$ to 1st order differential equation and solve.
3. Find the particular solution to $(D^4 - m^4)\ y = \sin\ mx$
4. Use variation of parameters to solve $y'' + y = \sec x$
5. Find the Fourier coefficient a_n for the function $f(x) = 1 + |x|$ defined in $-3 < x < 3$
6. Develop the Fourier Sine series of $f(x) = x$ in $0 < x < \pi$
7. Obtain the partial differential equation by eliminating arbitrary function from $x^2 + y^2 + z^2 = f(xy)$
8. Solve $y^2zp + x^2zq = xy^2$
9. Solve $u_x + u_y = 0$ using method of separation of variables
10. A finite string of length L is fixed at both ends and is released from rest with a displacement $f(x)$. What are the initial and boundary conditions involved in this problem?
11. Write all the possible solutions of one-dimensional heat transfer equation
12. Find the steady state temperature distribution in a rod of length 30cm having the ends at 20^0 C and 80^0 C respectively.

PART B

Answer one full question from each module

Module -I

13. (a)Verify linear independence of $e^{-x}cosx$ and $e^{-x}sinx$ using Wronskian and hence solve the initial value problem $y'' + 2y' + 2y = 0,\ y(0) = 0,\ y'(0) = 15$

(b) Find the general solution of the equation $x^2y'' + xy' + (x^2 - 0.25)y = 0$

If $y_1 = \frac{cosx}{\sqrt{x}}$

OR

14. (a) Find a second order homogeneous linear ODE for which $x,\ xlogx$ are solutions and solve the IVP with $y(1) = 2,\ y'(1) = 4$.

(b) Solve the IVP $y'' - 4y' + 9y = 0,\ y(0) = 0, y'(0) = -8$

Module- II

15. (a) Solve $(D^2 - 2D + 5)y = e^{2x}\sin x$

(b) Solve $((x+1)^2\frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} = (2x+3)(2x+4)$

OR

16. (a) Solve $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$

(b)Solve $y'' - 4y' + 5y = \frac{e^{2x}}{\sin x}$ by method of variation of parameters

Module - III

17. (a)Find the Fourier series representation of $f(x) = x\ sinx$ periodic with period 2π, defined in $0 < x < 2\pi$

(b)Find the Fourier cosine series of $f(x) = \cos x\ ,\ 0 < x < {}^{\pi}/_{2}$

OR

18. (a)Find the Fourier series expansion of $f(x) = e^{-x}$ in $-c < x < c$

(b)Develop the Sine series representation of $f(x) = \begin{cases} x\ , & 0 < x < 2 \\ 4 - x\ , & 2 < x < 4 \end{cases}$

Module - IV

19. (a) Solve $(y + zx)p - (x + yz)q = x^2 - y^2$

(b)Find the differential equation of all spheres of fixed radius having their centres in XY −plane.

OR

20. (a)Solve $\left(D^2 - 2DD' - 15D'^2\right)z = 12xy$

(b)Find the particular integral of

$$\left(D^3 - 7DD'^2 - 6D'^3\right)z = \sin(x + 2y) + e^{3x+y}$$

Module – V

21. A tightly stretched homogeneous string of length 20cm with its fixed ends executes transverse vibrations. Motion starts with zero initial velocity by displacing the string into the form $f(x) = K(x^2 - x^3)$ .Find the deflection $u(x,t)$ at any time t.

OR

22. A tightly stretched string of length 'a' with fixed ends is initially in equilibrium position. Find the displacement $u(x,t)$ of the string if it is set vibrating by giving each of its points a velocity $v_0 sin^3\left(\frac{\pi x}{a}\right)$.

Module - VI

23. Find the temperature distribution in a rod of length 2m whose end points are maintained at temperature zero and the initial temperature$f(x) = 100(2x - x^2)$

OR

24. The temperatures at the ends of a bar of length l cm with insulated sides are 30^oC and 90^oC respectively until steady state conditions prevail. If the temperature at each end is then suddenly reduced to 0^oC and maintained so, find the temperature distribution at a distance x at time t.

Reg. No._____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SECOND SEMESTER B.TECH DEGREE EXAMINATION, MAY 2017

MA 102: DIFFERENTIAL EQUATIONS

Max. Marks: 100 Duration: 3Hours

PART A

Answer all questions. 3 marks each.

1. Solve the initial value problem $y^{II} - y = 0,\ y(0) = 4, y^{I}(0) = -2$
2. Show that e^{2x}, e^{3x} are linearly independent solutions of the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0 \ \ in\ -\infty < x < +\infty$. What is its general solution?
3. Solve $\frac{d^3y}{dx^3}$ -4$\frac{d^2y}{dx^2} + 5\frac{dy}{dx}$ -2y =0
4. Find the particular integral of $(D^2 + 4D + 1)y = e^x sin3x$
5. Find the Fourier series of f(x)=x , $-\pi \le x \le \pi$
6. Obtain the half range cosine series of f(x)= x^2 , $0 \le x \le C$
7. Form the partial differential equation from $z = xg(y) + yf(x)$
8. Solve $(y - z)p + (x - y)q = (z - x)$
9. Write down the important assumption when derive one dimensional wave equation.
10. Solve $3u_x + 2u_y$=0 with u(x,0)=$4e^{-x}$ by the method of separation of variables.
11. Solve one dimensional heat equation when k> 0
12. Write down the possible solutions of one dimensional heat equation.

PART B

Answer six questions, one full question from each module.

Module I

13. a) Solve the initial value problem $y^{II} - 4y^1 + 13y = 0$ with y(0)= -1 ,$y^1(0) = 2$ **(6)**

    b) Solve the boundary value problem$y^{II} - 10y^I + 25y = 0$, y(0)=1,y(1)=0 **(5)**

OR

14. a) Show that $y_1(x) = e^{-4x}$ and $y_2(x) = xe^{-4x}$ are solutions of the differential equation $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y = 0$. Are they linearly independent? **(6)**

    b) Find the general solution of $(D^4 + 3D^2 - 4)y = 0$. **(5)**

Module II

15. a) Solve $(D^3 + 8\ )y = sinx \cos x + e^{-2x}$ **(6)**

    b) Solve $y^{II} + y = tan\ x$ by the method of variation of parameters. **(5)**

OR

16. a) Solve $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d2y}{dx2} + 2y = \frac{1}{x}$ **(6)**

b) Solve $(D^2 + 2D - 3)y = e^x \cos x$ **(5)**

Module III

17. a) Find the Fourier series of $f(x) = \begin{cases} -1 + x, & -\pi < x < 0 \\ 1 + x, & 0 < x < \pi \end{cases}$ **(6)**

b) Find the half range sine series of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \end{cases}$ **(5)**

OR

18. a) Obtain the Fourier series of $f(x) = \begin{cases} -\frac{\pi}{4}, & -\pi < x < 0 \\ \pi/4, & 0 < x < \pi \end{cases}$ **(6)**

b) Find the half range cosine series of $f(x) = x, 0 < x < l$ **(5)**

Module IV

19. a) Solve $(D^2 - 2DD' + D'^2)z = e^{x+2y} + x^3$ **(6)**

b) Find the Particular Integral of $\frac{\partial^3 z}{\partial x^3} - 7\frac{\partial^3 z}{\partial x^2 \partial y} - 6\frac{\partial^3 z}{\partial y^3} = \sin(x + 2y)$ **(5)**

OR

20. a) Solve $(D^2 + DD' - 6D'^2)z = y \sin x$ **(6)**

b) Solve $(m z - ny)p + (nx - lz)q = ly - m x$ **(5)**

Module V

21. Solve the one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(0,t) = 0, u(l,t) = 0$ *for* all t and initial conditions $u(x,0) = f(x), \left\{\frac{\partial u}{\partial t}\right\}_{t=0} = g(x)$. **(10)**

OR

22. A sting of length 20cm fixed at both ends is displaced from its position of equilibrium, by each of its points an initial velocity given by $= \begin{cases} x, & 0 < x \leq 10 \\ 20 - x, & 10 \leq x \leq 20 \end{cases}$, x being the distance from one end. Determine the displacement at any subsequent time. **(10)**

Module VI

23. Derive one-dimensional heat equation. **(10)**

OR

24. Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature 0°C, assuming that the initial temperature is $f(x) = \begin{cases} x, & 0 < x < L/2 \\ L - x, & L/2 < x < L \end{cases}$ **(10)**

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Total Pages: 2

Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SECOND SEMESTER B.TECH DEGREE EXAMINATION, JULY 2017

Course Code: MA102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions. Each carries3 marks.

1		Find a second order homogeneous linear ODE for which e^{-x} and e^{-2x} are the solutions.	(3)
2		Find a basis of solutions of $y^{11} - y^{1} = 0$.	(3)
3		Find the particular integral of $(D^2 - 4)y = x^2$.	(3)
4		Solve $(D^2 + 3D + 2)y = 5$.	(3)
5		Expand $\pi x - x^2$ in a half range sine series in the interval $(0, \pi)$.	(3)
6		Expand f(x) in Fourier series in the interval (-2 , 2) when $f(x) = \begin{cases} 0 & -2 < x < 0 \\ 1 & 0 < x < 2 \end{cases}$	(3)
7		Obtain the partial differential equation by eliminating the arbitrary function from $z = f(x^2 + y^2)$.	(3)
8		Solve $xp + yq = 3z$.	(3)
9		Using the method of separation of variables solve $u_{xy} - u = 0$.	(3)
10		Write down the possible solutions of one dimensional wave equation.	(3)
11		Find the solution of one dimensional heat equation in steady state condition.	(3)
12		State one dimensional heat equation with boundary conditions and initial conditions for solving it.	(3)

PART B

Answer six questions,one full question from each module.

Module 1

13	a)	Reduce to first order and solve $x^2y^{11} - 5xy^{1} + 9y = 0$.Given $y_1 = x^3$ is a solution.	(6)
	b)	Solve the initial value problem $4y^{11} - 25y = 0$ where $y(0) = 0$, $y^{1}(0) = -5$.	(5)

OR

14	a)	Show that the functions $e^{-x}\text{Cos}x$ and $e^{-x}\text{Sin}x$ are linearly independent. Form a second order linear ODE having these functions as solutions.	(6)
	b)	Solve $y^{1V} - 2y^{111} + 5y^{11} - 8y^{1} + 4y = 0$.	(5)

Module 1I

15	a)	Solve $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$.	(6)
	b)	Solve $y^{11} - 4y^{1} + 3y = e^{x}\text{Cos}\,2x$.	(5)

OR

16	a)	Solve $y^{11} + y = \text{Cosec}\,x$ using the method of variation of parameters.	(6)
	b)	Solve $(D^2 - 2D + 1)y = x\text{Sin}x$.	(5)

Module 1II

17 a) If $f(x) = x + x^2$ for $-\pi < x < \pi$ find the Fourier series expansion of f(x). (6)

b) Express $f(x) = |x|$ $-\pi < x < \pi$ as Fourier series. (5)

OR

18 a) Obtain Fourier series for the function $f(x) = \begin{cases} \pi x & when\ 0 \le x \le 1 \\ \pi(2-x) & when\ 1 \le x \le 2 \end{cases}$ (6)

b) Obtain the half range cosine series for f(x) = x in the 2interval $0 \le x \le \pi$. (5)

Hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots\ldots\ldots\ldots = \frac{\pi^2}{8}$.

Module 1V

19 a) Solve $xp - yq = y^2 - x^2$. (6)

b) Solve $\frac{\partial^2 z}{\partial x^2} - 7\frac{\partial^2 z}{\partial x \partial y} + 12\frac{\partial^2 z}{\partial y^2} = e^{x-y}$. (5)

OR

20 a) Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = SinxCos2y$. (6)

b) Solve $p - 2q = 3x^2 Sin(y + 2x)$. (5)

Module V

21 Derive one dimensional wave equation. (10)

OR

22 a) A tightly stretched homogeneous string of length l with its fixed ends at x = 0 and x = l executes transverse vibrations. Motion starts with zero initial velocity by displacing the string into the form $f(x) = k(x^2 - x^3)$.Find the deflection u(x,t) at any time t. (10)

Module VI

23 Find the temperature distribution in a rod of length 2m whose end points are maintained at temperature zero and the initial temperature is $f(x) = 100(2x - x^2)$. (10)

OR

24 A long iron rod with insulated lateral surface has its left end maintained at a temperature 0^oC and its right end at x=2 maintained at 100^0C. Determine the temperature as a function of x and t if the initial temperature is (10)

$u(x,0) = \begin{cases} 100x & 0 < x < 1 \\ 100 & 1 < x < 2 \end{cases}$.

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Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SECOND SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

Course Code: MA102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks. Marks

1 Solve the initial value problem $xy' = y - 1, y(0) = 1$ (3)

2 Solve the following differential equation by reducing it to first order $xy" = 2y'$. (3)

3 Find the particular integral of $(D^2 + 3D + 2)y = 3$. (3)

4 Find the particular integral of $y" + y = \sin x$. (3)

5 Obtain the Fourier series expansion for the function $f(x) = x$ in the range $-\pi < x < \pi$. (3)

6 Find the Fourier sine series of the function $f(x) = \pi x - x^2$ in the interval $(0, \pi)$ (3)

7 Form a partial differential equation by eliminating the arbitrary function in $xyz = \emptyset(x + y + z)$ (3)

8 Solve $r + s - 2t = e^{x+y}$. (3)

9 Solve one dimensional wave equation for $k < 0$. (3)

10 Solve $\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} - u = 0, u(x, 0) = 6e^{-3x}$ using method of separation of variables. (3)

11 Find the steady state temperature distribution in a rod of length 30cm if the ends are kept at 20^0C and 80^0C. (3)

12 Write down the possible solutions of one dimensional heat equation. (3)

PART B

Answer six questions, one full question from each module.

Module I

13 a) Verify that the given functions $x^{\frac{3}{2}}, x^{-\frac{1}{2}}$ are linearly independent and form a basis of solution space of given ODE $4x^2y" - 3y = 0$. (6)

b) Solve the boundary value problem: (5)

$$y'' - 10y' + 25y = 0, \quad y(0) = 1, \quad y(1) = 0.$$

OR

14 a) Find the general solution of $y'''' + 2y'' + y = 0$. (6)

b) Find a fundamental set of solutions of $2t^2y'' + 3ty' - y = 0, t < 0$. Given that $y_1(t) = \frac{1}{t}$ is a solution. (5)

Module II

15 a) Find the particular integral of $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4cos^2x$. (6)

b) Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$ using method of variation of parameters. (5)

OR

16 a) Solve $x^2y'' - xy' - 3y = x^2 \ln x$ (6)

b) Solve $y'''' - 2y''' + 5y'' - 8y' + 4y = e^x$. (5)

Module III

17 a) Obtain the Fourier series expansion of $f(x) = x \sin x$ in the interval $(-\pi, \pi)$. (6)

b) Find the half range sine series of $f(x) = k$ in the interval $(0, \pi)$. (5)

OR

18 a) Find the Fourier series of $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval $(0, 2\pi)$. (6)

b) Find the half range sine series of $f(x) = e^x$ in $(0,1)$. (5)

Module IV

19 a) Solve $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 2\sin(3x + 2y)$ (6)

b) Solve $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$ (5)

OR

20 a) Form the PDE by eliminating a, b, c from $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ (6)

b) Solve $(x + y)zp + (x - y)zq = x^2 + y^2$. (5)

Module V

21 A tightly stretched violin string of length 'a' and fixed at both ends is plucked at its mid-point and assumes initially the shape of a triangle of height 'h'. Find the displacement u(x,t) at any distance 'x' and any time 't' after the string is released from rest. (10)

OR

22 Solve the PDE $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$. (10)

Boundary conditions are $u(0, t) = u(l, t) = 0,\ t \geq 0$

Initial conditions are $y(x, 0) = a \sin\left(\frac{\pi x}{l}\right)$ and $\frac{\partial y}{\partial t} = 0 \quad at \quad t = 0$.

Module VI

23 A rod, 30 cm long has its ends A and B kept at 20^0C and 80^0C respectively, until the steady state conditions prevail. The temperature at each end is then suddenly reduced to 0^0C and kept so. Find the resulting temperature function u(x,t) taking x=0 at A. (10)

OR

24 A long iron rod with insulated lateral surface has its left end maintained at a temperature 0^0C and its right end at x=2, maintained at 100^0C. Determine the temperature as a function of 'x' and 't' if the initial temperature is (10)

$$u(x, 0) = \begin{cases} 100x, & 0 < x < 1 \\ 100, & 1 < x < 2 \end{cases}.$$

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Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SECOND SEMESTER B.TECH DEGREE EXAMINATION, JULY 2018

Course Code: MA102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks Marks

1 Consider the initial value problem $y''-x^3y'+6xy=\sin x,\ y(0)=3, y'(0)=-1$. Can this problem have unique solution in an interval containing zero? Explain. (3)

2 Find any three independent solutions of the differential equation $y'''-y'=0$. (3)

3 Find the particular solution of the differential equation $y''-6y'+9y=e^{3x}$. (3)

4 Using a suitable transformation, convert the differential equation $(2x-3)^2y''-(2x-3)y'+2y=(2x-3)^2$ into a linear differential equation with constant coefficients. (3)

5 State the conditions for which a function f(x) can be represented as a Fourier series. (3)

6 Discuss the convergence of a Fourier series of a periodic function f(x) of period 2π. (3)

7 Find the partial differential equation representing the family of spheres whose centers lies on z-axis. (3)

8 Find the particular solution of $(D^2-2DD'+2D'^2)z=\sin(x-y)$ (3)

9 Write any three assumptions involved in the derivation of one dimensional wave equation. (3)

10 A string of length *l* fastened at both ends. The midpoint of the string is taken to a height *h* and then released from rest in that position. Write the boundary conditions and initial conditions of the string to find the displacement function y(x,t) satisfying the one dimensional wave equation. (3)

11 Write the fundamental postulates used in the derivation of one dimensional heat equation. (3)

12 Define steady state condition in one dimensional heat equation $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$. (3)

PART B

Answer six questions,one full question from each module

Module 1

13 a) Discuss the existence and uniqueness of solution of the initial value problem $\frac{dy}{dx}=\frac{y}{\sqrt{x}}, y(1)=3$. (6)

b) Prove that $y_1(x)=e^x$ and $y_2(x)=e^{4x}$ form a fundamental system(basis) for the (5)

differential equation $y''-5y'+4y=0$. Can $5e^{x}-2e^{4x}$ be a solution(do not use verification method) of the differential equation? Explain.

OR

14 a) Discuss the existence and uniqueness of solution of the initial value problem (6)

$$\frac{dy}{dx}=x^2+y^2\ ,y(0)=1 \text{ in the rectangle } |x|\le 1, |y-1|\le 1.$$

b) If $y_1(x)=x$ is a solution of $x^2y''+2xy'-2y=0$, find the general solution. (5)

Module II

15 a) By the method of variation of parameters, solve $y''+y=x\sin x$. (6)

b) Solve $y''+5y'+6y=e^{-2x}\sin 2x$. (5)

OR

16 a) Solve $x^2y''+xy'-9y=\log x$. (6)

b) Solve $y''-2y'+5y=x^2$. (5)

Module III

17 Find the Fourier cosine series representation of $f(x)=x, 0\le x\le \pi$. Also find the Fourier series representation f(x) if f(x) is periodic function with period π. (11)

OR

18 Find the Fourier series of the periodic function f(x) of period 4, where (11)

$f(x)=\begin{cases}2\ , & -2<x\le 0\\ x\ , & 0<x<2\end{cases}$ and deduce that

$$(i)\ \ 1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+....=\frac{\pi^2}{8} \text{ and } (ii)\ \ 1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+....=\frac{\pi}{4}$$

Module IV

19 a) Find the particular solution of $\frac{\partial^2 z}{\partial x^2}+3\frac{\partial^2 z}{\partial x\partial y}+2\frac{\partial^2 z}{\partial y^2}=y^2$. (5)

b) Find the general solution of $(y^2+z^2)p-xyq=-xz$. (6)

OR

20 a) Solve $(D^2+3DD'+2D'^2)z=(2x+y)^7$. (5)

b) Solve $4\frac{\partial^2 z}{\partial x^2}-4\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}=16\log(x+2y)$. (6)

Module V

21 a) Using method of separation of variables, solve $\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}-u$, $u(x,0)=5e^{-3x}$. (5)

b) A tightly stretched string of length *l* fastened at both ends is initially in a position given by y = kx, 0 < x < l. If it is released from rest from this position, find the displacement y(x,t) at any time t and any distance x from the end x = 0. (5)

OR

22 A string is stretched and fastened in two points 50 cm apart. Motion is started by (10)

displacing the string into the form of the curve y = x(50 - x) and also by imparting a constant velocity V to every point of the string in the position at time t = 0. Determine the displacement function y(x, t).

Module VI

23 A rod of length 50 cm has its ends A and B kept at 20^0C and 70^0Crespectively until steady state temperature prevail. The temperature at each end is thensuddenly reduced to zero temperature and kept so. Find the resulting temperature function u(x,t) taking x = 0 at A. (10)

OR

24 A bar 10 cm long with insulated sides has its ends A and B maintainedat 50^0C and 100^0C respectively until steady state conditions prevail. The temperature atA is suddenly raised to 90^0C and at the same time that at B is lowered to 60^0C. Find thetemperature distribution in the bar at time t. (10)

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Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SECOND SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2018

Course Code: MA102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks

1	Find a general solution of the ordinary differential equation $y'' + y = 0$	(3)
2	Find the Wronskian of $e^x \cos 2x$ and $e^x \sin 2x$	(3)
3	Find the particular integral of the differential equation $y'' + y = cosh5x$	(3)
4	Using a suitable transformation, convert the differential equation. $(3x+2)^2y'' + 5(3x+2)y' - 3y = x^2 + x + 1$ into a linear differential equation with constant coefficients.	(3)
5	If $f(x)$ is a periodic function of period $2L$ defined in $[-L, L]$. Write down Euler's Formulas $a_0,\ a_n,\ b_n$ for $f(x)$.	(3)
6	Find the Fourier cosine series of f(x) = x^2 in $0<x\leq c$.	(3)
7	Find the partial differential equation of all spheres of fixed radius having their centres in xy-plane.	(3)
8	Find the particular integral of r + s - 2t =$\sqrt{2x+y}$.	(3)
9	Write any three assumptions involved in the derivation of one dimensional wave equation.	(3)
10	Solve $x\frac{\partial u}{\partial x} - 2y\frac{\partial u}{\partial y} = 0$ using method of separation of variables.	(3)
11	Find the steady state temperature distribution in a rod of 30 cm having its ends at 20^0C and 80^0C respectively.	(3)
12	Write down the possible solutions of the one dimensional heat equation.	(3)

PART B

Answer six questions, one full question from each module

Module 1

13	a)	Solve the initial value problem $y'' + 4y' + 5y = 0, y(0) = 2, y'(0) = -5$.	(5)
	b)	Find a basis of solutions of the ODE $(x^2 - x)y'' - xy' + y = 0$, if y_1= x is a	(6)

solution.

OR

14 a) Reduce to first order and solve $y'' + (1 + \frac{1}{y})(y')^2 = 0$ (5)

b) Solve the initial value problem $9y'' - 30y' + 25y = 0, y(0) = 3, y'(0) = 10.$ (6)

Module 1I

15 a) Solve $y'' - 2y' + 5y = e^{2x} sinx.$ (5)

b) Using method variation of parameters solve $y'' + 4y = tan2x$ (6)

OR

16 a) Solve $x^3y''' + 3x^2y'' + xy' + y = x + logx$ (5)

b) Solve using method of variation of parameters $y'' - 2y' + y = \frac{e^x}{x}$ (6)

Module 1II

17 Find the Fourier series of periodic function $f(x) = \begin{cases} -x, -1 \le x \le 0 \\ x, 0 \le x \le 1 \end{cases}$ with period 2. Hence prove that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots = \frac{\pi^2}{8}$. (11)

OR

18 Find the Fourier series of periodic function $f(x) = xsinx$,0 < x < 2 π with period 2 π. (11)

Module 1V

19 a) Solve p - 2q = $3x^2$ sin(y+2x). (5)

b) Solve r + s - 6t = y sinx. (6)

OR

20 a) Solve $x(y - z)p + y(z - x)q = z(x - y)$. (5)

b) Solve $(D^2 - 2DD' - 15D'^2)$ z = 12xy. (6)

Module V

21 A tightly stretched string of length L is fixed at both ends. Find the displacement u(x,t) if the string is given an initial displacement f(x) and an initial velocity g(x). (10)

OR

22 A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $u = v_0 sin^3\left(\frac{\pi x}{l}\right), 0 \le x \le l$. If it is released from rest from this position, find the displacement function $u(x, t)$ (10)

Module VI

23 The ends A and B of a rod of length L are maintained at temperatures 0^0C and 100^0C respectively until steady state conditions prevails. Suddenly the temperature at the end A is increased to 20^0C and the end B is decreased to 60^0C. Find the temperature distribution in the rod at time t. (10)

OR

24 Find the temperature distribution in a rod of length 2 m whose end points are maintained at temperature zero and the initial temperature is (10)

$f(x) = 100(2x-x^2)$, $0 \leq x \leq 2$

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Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SECOND SEMESTER B.TECH DEGREE EXAMINATION (R & S), MAY 2019

Course Code: MA102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks

1	Find the general solution of $\frac{d^3y}{dx^3} + y = 0$	(3)
2	Find the Wronskian of $e^x \cos 2x$ and $e^x \sin 2x$	(3)
3	Find the Particular Integral of . $y'' - 4y' - 5y = 4\,cos2x.$	(3)
4	Find the particular integral of $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = \sinh 2x$	(3)
5	Evaluate the coefficient a_n in the Fourier series expansion for $f(x) = \lvert\sin x\rvert$ in $-\pi < x < \pi$	(3)
6	Find the half range Fourier sine series representation of $f(x) = k$ in $(0,\pi)$	(3)
7	Find the partial differential equation of all spheres having their centre lies on z-axis.	(3)
8	Form the partial differential equation of $z = f(\frac{xy}{z})$ by eliminating the arbitrary function f.	(3)
9	Solve $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}, u(0,y) = 8e^{-3y}$, using the method of separation of variables.	(3)
10	A tightly stretched string of length l is fixed at both ends and pulled from its mid point to a height h and realised from rest from this position. Write down the initial and boundary conditions.	(3)
11	Find the steady state temperature distribution in a rod of length 30 cm, if the ends of the rod are kept at $20^\circ C$ and $80^\circ C$	(3)
12	Write down the three possible solutions of the one dimensional heat equation.	(3)

PART B

Answer six questions,one full question from each module

Module 1

13 a) Solve the initial value problem $y'' + 4y' + 5y = 0, y(0) = 2, y'(0) = -5.$ (6)

b) Find the general solution of the differential equation $y''' - y'' + 4y' = 0$ (5)

OR

14 a) If $y_1(x) = x$ is a solution to the differential equation $(1 + x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$, find the general solution. (6)

b) Solve the ordinary differential equation $y''' - 3y'' - 4y' + 6y = 0.$ (5)

Module 1I

15 a) Solve $2(3x + 1)^2\frac{d^2y}{dx^2} + 21(3x + 1)\frac{dy}{dx} + 18y = 9x$ (6)

b) Solve $(D^4 + 2D^2 + 1)y = x^4$ (5)

OR

16 a) Use method variation of parameters to solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$ (6)

b) Solve $(D^2 - 4D + 4)y = sin^2$ x (5)

Module 1II

17 a) Obtain the half range Fourier cosine series expansion of $f(x) = x \sin x$ in (0,π) . (6)

b) Find the Fourier series for $f(x) = |x|$, -π<x<π (5)

OR

18 a) Find the Fourier series for $f(x) = \begin{cases} 0, -\pi < x < 0 \\ \pi, 0 < x < \pi \end{cases}$ (6)

b) Find the Fourier series of the periodic function f(x) of period 4, where $f(x) = \begin{cases} 0, -2 < x \leq -1 \\ k, -1 < x < 1 \\ 0, 1 \leq x < 2 \end{cases}$ (5)

Module 1V

19 a) Solve $\frac{y^2z}{x}p + xzq = y^2$ (6)

b) Find the partial differential equation of all planes which are at a constant distance k from the origin. (5)

OR

20 a) Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$ (6)

b) Solve $(D^2 + 3DD' + 2D'^2)z = x^2y^2$ (5)

Module V

21 A string is stretched between two fixed points at a distance of 60 cm and the points of the string are given initial velocities where

$$v = \begin{cases} \frac{\lambda x}{30}, & 0 < x < 30 \\ \frac{\lambda}{30}(60 - x), & 30 < x < 60 \end{cases}$$

, x being the distance from an end, find the displacement at any time t. (10)

OR

22 A uniform elastic string of length 60 cm is subjected to a constant tension of 2 Kg. If the ends are fixed, the initial displacement $u(x, 0) = 60x - x^2, 0 < x < 60$ and the initial velocity is zero, find the displacement function $u(x, t)$ (10)

Module VI

23 Find the temperature distribution in a rod of length 2m whose end points are maintained at temperature $0^\circ C$ and the initial temperature is $f(x) = 100(2x - x^2), 0 \leq x \leq 2$ (10)

OR

24 A bar 10 cm long with insulated sides has its ends A and B maintained at $50^\circ C$ and $100^\circ C$ respectively until steady state conditions prevail. The temperature of A is suddenly raised to $90^\circ C$ and at the same time that at B is lowered to $60^\circ C$. Find the temperature distribution in the bar at time t. (10)

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Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SECOND SEMESTER B.TECH DEGREE EXAMINATION(S), DECEMBER 2019

Course Code: MA102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks

1		Find a general solution of the ordinary differential equation $y'' + y = 0$	(3)
2		Reduce to first order and solve. $yy'' = 3(y')^2$.	(3)
3		Find the particular integral of $y'' - 4y' - 5y = 4\,cos2x.$	(3)
4		Using a suitable transformation, convert the differential equation $(x^2D^2 + xD + 1)y = logx$ into a linear differential equation with constant coefficients.	(3)
5		If $f(x)$ is a periodic function of period 2π defined in $[-\pi, \pi]$. Write down Euler's Formulas $a_0,\ a_n,\ b_n$ for $f(x)$.	(3)
6		Find the half range Fourier cosine series of the function $f(x) = x$ in the range $0 < x < 2.$	(3)
7		Find the PDE by eliminating arbitrary function φ from $xyz = \varphi(x + y + z)$.	(3)
8		Solve $(D + 2D')(D - 3D')^2 z = 0.$	(3)
9		Write any three assumptions involved in the derivation of one dimensional wave Equation.	(3)
10		A tightly stretched string of length l is fixed at both ends and pulled from its mid point to a height h and released from rest from this position. Write down the initial and boundary conditions.	(3)
11		Write all possible solutions of one dimensional heat equation.	(3)
12		Find the steady state temperature distribution in a rod of length l if the ends are kept at 0^0C and 100^0C.	(3)

PART B

Answer six questions,one full question from each module

Module 1

13	a)	Solve $y'' - 2y' + y = 0,\ y(0) = 1,\ y'(0) = 2.$	(6)
	b)	Find a basis of solutions of the ODE $(x^2 - x)y'' - xy' + y = 0$, if y_1= x is a	(5)

solution.

OR

14 a) Solve the ordinary differential equation $y''' - 3y'' - 4y' + 6y = 0$. (6)

b) Solve the ordinary differential equation $xy'' + 2y' + xy = 0$, given that $y_1 = \frac{sinx}{x}$ is a solution. (5)

Module 1I

15 a) By the method of variation of parameters, solve $y'' + 4y = tan2x$. (6)

b) Solve $y'' + 2y = x^2e^{3x}$. (5)

OR

16 a) Solve $(x+3)^2y'' - 4(x+3)y' + 6y = 3x$. (6)

b) Solve $x^2y'' - 4xy' + 6y = x^5$. (5)

Module 1II

17 a) Find the Fourier series of f defined by $f(x) = x - x^2$ in $(-1,1)$. (6)

b) Expand $f(x) = c$ in the half range sine-series in $0 \leq x \leq \pi$. (5)

OR

18 Obtain Fourier series for the function $f(x) = |cosx|, -\pi \leq x \leq \pi$. (11)

Module 1V

19 a) Solve $r + s + 2t = e^{x+y}$. (6)

b) Find the general solution of $x^2(y-z)p + y^2(z-x)q = (x-y)z^2$. (5)

OR

20 a) Solve $(D^3 + D^2D' - DD'^2 - D'^3)z = e^x \cos 2y$ (6)

b) Solve $(D^2 + 3DD' + 2D'^2)z = x^2y^2$ (5)

Module V

21 A uniform elastic string of length 60 cm is subjected to a constant tension of 2 Kg. If the ends are fixed, the initial displacement $u(x,0) = 60x - x^2, 0 < x < 60$ and the initial velocity is zero, find the displacement function $u(x,t)$ (10)

OR

22 Find the deflection of the vibrating string which is fixed at the ends $x = 0$ and $x = 2$ and the motion is started by displacing the string into the form $\sin^3(\frac{\pi x}{2})$ and released it with zero initial velocity at $t = 0$. (10)

Module VI

23 Find the temperature distribution in a rod of length $2m$ whose endpoints are maintained at temperature zero and initial temperature is $f(x) = 100(2x - x^2)$. (10)

OR

24 A rod of length 30cm has its ends A and B kept at 20^0C and 80^0C respectively until steady state temperature prevails. Suddenly the temperature at A is raised to 60^0C and the end B is decreased to 40^0C . Find the temperature distribution in the rod at time t. (10)

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Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech S1,S2 (S) Examination September 2020 (2015 Scheme)

Course Code: MA102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks

1	Find the ODE $y'' + ay' + by = 0$ for the basis $\{e^x, xe^x\}$	(3)
2	Reduce to first order and solve $2xy'' = 3y'$.	(3)
3	Find the particular integral of $y'' + 4y' + 4y = x^2$.	(3)
4	Using a suitable transformation, convert the differential equation $(x^2D^2 - 4xD + 6)y = x$ into a linear differential equation with constant coefficients.	(3)
5	If $f(x)$ is a periodic function of period $2L$ defined in $[-L, L]$. Write down Euler's Formulas a_0, a_n, b_n for $f(x)$.	(3)
`6	Find the Fourier series of the function $f(x) = x$ in the range $-\pi < x < \pi$.	(3)
7	Find the PDE by eliminating arbitrary constants a and b from $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.	(3)
8	Find the particular integral of $(D^3 - 4D^2D' + 4DD'^2)z = \cos(2x + y)$.	(3)
9	Write all possible solutions of one dimensional wave equation.	(3)
10	A homogeneous string is stretched and its ends are fixed at $x = 0$ and $x = 40$. Motion is started by displacing the string into the form $f(x) = \sin\left(\frac{\pi x}{40}\right)$ from which it is released at time $t = 0$. Write the boundary and initial conditions..	(3)
11	Solve one dimensional heat equation for $\lambda < 0$.	(3)
12	Find the steady state temperature distribution in a rod of length 40 cm if the ends are kept at 0^0C and 100^0C.	(3)

PART B

Answer six questions, one full question from each module

Module 1

13 a) Solve $y'' - 2y' - 3y = 0, y(-1) = e, y'(1) = -\frac{e}{4}$. (6)

b) Show that the functions x^3 and x^5 are the basis of solutions of ODE $x^2y'' - 7xy' + 15y = 0$. (5)

OR

14 a) Solve ODE $y^v - 3y^{iv} + 3y''' - y'' = 0$. (6)

b) Solve the ODE $xy'' + 2y' + xy = 0$. Given that $y_1 = \frac{cosx}{x}$ is a solution. (5)

Module 1I

15 a) By the method of variation of parameters, solve $y'' + y = secx$. (6)

b) Solve $x^2y'' - 4xy' + 6y = x^5$. (5)

OR

16 a) Solve $(2x+3)^2y'' - 2(2x+3)y' - 12y = 6x$. (6)

b) Solve $y'' + 2y' - 3y = e^{2x}sinx$. (5)

Module 1II

17 a) Find the Fourier series of f defined by $f(x) = e^x$ in $(-\pi, \pi)$. (11)

OR

18 a) Obtain Fourier series for the function $f(x) = x^2, -\pi \leq x \leq \pi$. (6)

b) Expand $f(x) = cosx$ as a half range sine-series in $0 \leq x \leq \pi$. (5)

Module 1V

19 a) Solve $r + s - 2t = \sqrt{2x + y}$. (6)

b) Find the general solution of $x^2p + y^2q = (x+y)z$. (5)

OR

20 a) Solve $4r + 12s + 9t = e^{3x-2y}$. (6)

b) Solve $(D^2 - DD' - 6D'^2)z = xy$. (5)

Module V

21 a) Using method of separation of variables, solve $y^2u_x - x^2u_y = 0$. (5)

b) Find the displacement of a finite string of length *l* that is fixed at both ends and is released from rest with an initial displacement of $2\sin\left(\frac{\pi x}{l}\right)$. (5)

OR

22 Derive one dimensional wave equation. (10)

Module VI

23 A rod of length L is heated so that its ends A and B are at zero temperature .If its initial temperature is given by $u = \frac{cx(L-x)}{L^2}$, find the temperature at time t. (10)

OR

24 A rod of length 40cm has its ends A and B kept at 0^0C and 100^0C respectively until steady state conditions prevail. Suddenly the temperature at A is raised to 20^0C and the end B is decreased to 60^0C . Find the temperature distribution in the rod at time t. (10)

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