

Reg. No.: _____

Name: _____

FIRST SEMESTER B.TECH DEGREE EXAMINATION, JANUARY 2016

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each question carries 3 marks

1. Show that the series $\sum_{k=1}^{\infty} \frac{\cos k}{k^2}$ is convergent.
2. Find $\frac{d}{dx} (e^x \operatorname{sech}^{-1} \sqrt{x})$
3. Identify the surfaces $5x^2 - 4y^2 + 20z^2 = 0$
4. Equation of a surface in spherical coordinates is $\rho = \sin \theta \sin \phi$
Find the equation of this surface in rectangular coordinates.
5. Given $f = e^x \sin y$; show that the function satisfies the Laplace equation $f_{xx} + f_{yy} = 0$
6. Let $w = 4x^2 + 4y^2 + z^2$, where $x = \rho \sin \phi \cos \theta$, $y = \rho \sin \phi \sin \theta$, $z = \rho \cos \phi$ Find $\frac{\partial w}{\partial \rho}$ using chain rule.
7. A particle moves along a circular helix in 3-space so that its position vector at time t is $r(t) = (4 \cos \pi t)i + (4 \sin \pi t)j + tk$ Find the displacement of the particle during the interval $1 \leq t \leq 5$.
8. Find the tangent to the curve $r(t) = (t^2 - 1)i + t^2j$ at $t = 1$
9. Evaluate $\int_1^a \int_1^b \frac{dy dx}{xy}$
10. The line $y = 2 - x$ and the parabola $y = x^2$ intersect at the points $(-2, 4)$ and $(1, 1)$. If R is the region enclosed by $y = 2 - x$ and $y = x^2$, then find $\iint_R (y) dA$

(10 x 3 = 30 Marks)

PART B

Answer any 2 complete questions each having 7 marks

11. Find the radius of convergence and interval of convergence of the series $\sum_{k=1}^{\infty} \frac{(x-5)^k}{k^2}$.
12. Test the convergence of $\frac{x}{12} + \frac{x^2}{23} + \frac{x^3}{34} + \dots$
13. Find the Taylor's series of $\frac{1}{x}$ about $x = 1$.

Answer any 2 complete questions each having 7 marks

14. Find the domains of (i) $f(x, y) = \sqrt{25 - x^2 - y^2 - z^2}$ (ii) $f(x, y) = \ln(x - y^2)$ and describe them in words.
15. Find the limit of $f(x, y) = \frac{-xy}{x^2 + y^2}$ as $(x, y) \rightarrow (0, 0)$ along (i) the X-axis, (ii) the Y-axis (iii) the line $y = x$.
16. Find the spherical and cylindrical coordinates of the point that has rectangular coordinates $(x, y, z) = (4, -4, 4\sqrt{6})$

Answer any 2 complete questions each having 7 marks

17. Locate all relative maxima, relative minima and saddle point if any, of $f(x, y) = y^2 + xy + 4y + 2x + 3$.
18. Let f be a differentiable function of 3 variables and suppose that $W = f(x - y, y - z, z - x)$. Prove that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$.
19. Find the local linear approximation $L(x, y)$ to $f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$ at the point $P(4, 3)$. Compare the error in approximating 'f' by L at the specified point $Q(3.92, 3.01)$ with the distance between P and Q.

Answer any 2 complete questions each having 7 marks

20. Find $y(t)$ where $y''(t) = 12t^2 \mathbf{i} - 2t \mathbf{j}$, $y(0) = 2\mathbf{i} - 4\mathbf{j}$, $y'(0) = 0$.
21. Find the arc length parametrization of the line $x = 1 + t, y = 3 - 2t, z = 4 + 2t$ that has the same direction as the given line and has reference point $(1, 3, 4)$.
22. Find the directional derivative of $f(x, y) = e^x \sec y$ at $P(0, \pi/4)$ in the direction of PQ where Q is the origin.

Answer any 2 complete questions each having 7 marks

23. Find the area bounded by the x-axis, $y = 2x$ and $x + y = 1$ using double integration.
24. Use a triple integral to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$.
25. Sketch the region of integration and evaluate the integral $\int_1^2 \int_y^{y^2} dx dy$ by changing the order of integration.

10103

Reg. No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION, JUNE 2016

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer ALL questions. Each question carries 3 marks

1. Evaluate $\int_0^1 \sinh^2(2x) dx$
2. Check whether the series $\sum_{k=1}^{\infty} \frac{1}{2k-1}$ converges or not.
3. Identify the quadric surface $6x^2 + 3y^2 + 4z^2 = 24$
4. Convert $(2\sqrt{3}, \pi/3, 6)$ from cylindrical to spherical co-ordinates.
5. Find the rate of change of $f(x,y) = xe^y + 5y$ with respect to x at the point $(4,0)$ with y held fixed.
6. If $f(x,y) = x^2y^3 + x^4y$. Find f_{xy}
7. Evaluate $\int_1^9 \left(\left(\frac{t}{2}\right) i + \left(t - \frac{1}{2}\right) j \right) dt$
8. Find $\frac{d\vec{u}}{dt}$ if $\vec{u}(t) = (3t i + 5 t^2 j + 6 k) \cdot (t^2 i + 2t j + t k)$
9. Sketch the region of integration in $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dy dx$
10. Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{(x+y+z)} dx dy dz$

PART B*Answer any 2 complete questions each having 7 marks*

11. A ball is dropped from a height of h feet and on each bounce rises 75% of the distance it has fallen previously. If it travels a distance of 21 feet what is h ?

12. Use Ratio Test for absolute convergence to find whether the series

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1} 2^k}{k!} \text{ converges.}$$

13. Find the Maclaurin's Series for $\frac{1}{1-x}$

Answer any 2 complete questions each having 7 marks

14. For the surface $4x^2 + 9y^2 + 18z^2 = 72$

a. Find the equation of the elliptical trace in the plane

b. $z = \sqrt{2}$

c. Find the length of the major and minor axes of the ellipse.

15. Find $\lim_{(x,y) \rightarrow (0,0)} (x^2 + y^2) \ln(x^2 + y^2)$

16. Let $f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$. Determine the limit of $f(x, y)$ as

(x, y) approaches $(0, 0)$ along the curve C . where C is

(a) $x = 0$ (b) $y = 0$ (c) $y = x$

(d) $y = x^2$ (e) $x = y^2$

Answer any 2 complete questions each having 7 marks

17. Use chain rule to find $\frac{dw}{ds}$ at $s = 1/4$ if

$$w = r^2 - r \tan \theta; \quad r = \sqrt{s}; \quad \theta = \pi s$$

18. Locate all relative extrema and saddle points of $f(x,y) = x^2 + xy - 2y - 3x + 1$

19. The volume V of a right circular cone of radius r and height h is given by $V =$

$$\frac{1}{3} \pi r^2 h. \text{ Suppose that the height decreases from 20 to 19.95 units and the radius}$$

increases from 4 to 4.05 units. Compare the change in volume of the cone with an approximation of this change using a total differential.

Answer any 2 complete questions each having 7 marks

20. The temperature in degree Celsius at a point in the (x, y) plane is

$$T(x, y) = \frac{xy}{1 + x^2 + y^2}$$

Find the rate of change of temperature at $(1, 1)$ in the direction of $(2\hat{i} - \hat{j})$.

21. Find the scalar tangential and normal components of acceleration at time t of a

particle with position vector at time t is $\mathbf{r}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}$

22. Find the equation of the tangent plane and parametric equation for the normal

line to the surface $x^2 + y^2 + z^2 = 25$ at $P(3, 0, 4)$

Answer any 2 complete questions each having 7 marks

23. Evaluate $\iint_R \sin \theta dA$ where R is the region in the first quadrant that is outside the

circle $r = 2$ and inside the cardioid $r = 2(1 + \cos \theta)$.

24. Find the Jacobian $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ where $x = 4u + v$, $y = u - 2w$,

$$z = v + w.$$

25. By changing the order of integration evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$

10105

Reg. No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE SPECIAL EXAMINATION, AUGUST 2016

Course Code: MA101**Course Name: CALCULUS**

Max. Marks: 100

Duration: 3 Hours

PART A*Answer ALL questions. Each question carries 3 marks*

1. Find derivative of $y = \sinh(4x-8)$
2. Test whether the series converges or diverges, $\sum_{k=1}^{\infty} \frac{k}{2^k}$
3. Identify the surface $z = y^2 - x^2$
4. Convert from rectangular to spherical co-ordinates, $(2\sqrt{3}, 2, -4)$
5. Find $\frac{\partial Z}{\partial x}$ and $\frac{\partial Z}{\partial y}$ if $Z = \cos(xy^3)$
6. Show that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ if $z = x^2y + 5y^3$.
7. Evaluate $\int_0^2 (2t\hat{i} + 3t^2\hat{j}) dt$
8. Find the arc length of the parametric curve $x=e^t, y=e^{-t}, z = \sqrt{2}t, 0 \leq t \leq 1$.
9. Evaluate $\int_1^3 \int_2^4 (40 - 20xy) dy dx$
10. Evaluate $\int_0^3 \int_0^2 \int_0^1 (xyz) dx dy dz$

PART B*Answer any 2 complete questions each having 7 marks*

11. Test the convergence of the series $\sum_{k=1}^{\infty} \frac{k(k+3)}{(k+1)(k+2)(k+5)}$
12. Show that $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$
13. Find the Taylor series of $\frac{1}{x+2}$ about $x=1$.

Answer any 2 complete questions each having 7 marks

14. Express the equation $x^2 - y^2 - z^2 = 0$ in cylindrical and spherical coordinates.

15. Evaluate $\lim_{(x,y) \rightarrow (0,0)} [\sin(\sqrt{x^2 + y^2})]/(x^2 + y^2)$ by converting to polar coordinates.
16. Show that the functions $f(x, y) = 3x^2y^5$ and $f(x, y) = \sin(3x^2y^5)$ are continuous everywhere.

Answer any 2 complete questions each having 7 marks

17. Let $L(x, y)$ denote the local linear approximation to $f(x, y) = \sqrt{x^2 + y^2}$ at the point $(3, 4)$. Compare the error in approximating $f(3.04, 3.98) = \sqrt{(3.04)^2 + (3.98)^2}$ by $L(3.04, 3.98)$ with the distance between the points $(3, 4)$ and $(3.04, 3.98)$.
18. Suppose that $w = x^2 + y^2 - z^2$ and $x = \rho \sin \phi \cos \theta$, $y = \rho \sin \phi \sin \theta$, $z = \rho \cos \phi$. Use appropriate form of the chain rule to find $\frac{\partial w}{\partial \rho}$ and $\frac{\partial w}{\partial \theta}$.
19. Locate the relative extrema and saddle points of $f(x, y) = 3x^2 - 2xy + y^2 - 8y$

Answer any 2 complete questions each having 7 marks

20. Let $f(x, y) = x^2e^y$. Find the maximum value of a directional derivative at $(-2, 0)$ and find the unit vector in the direction in which the maximum value occur.
21. Find the angle between the tangent lines to the graphs of $r_1(t) = \tan^{-1}t i + \sin t j + t^2 k$
 $r_2(t) = (t^2 - t)i + (2t - 2)j + \log t k$
22. Suppose that a particle moves through 3-space so that its position vector at time t is $r(t) = ti + t^2j + t^3k$. Find the scalar tangential and normal components of acceleration at time $t = 1$.

Answer any 2 complete questions each having 7 marks

23. Use a polar double integral to find the area enclosed by the circle $r = \sin \theta$
24. Use a triple integral to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $z = 5$
25. Evaluate $\iint_R \frac{x-y}{x+y} dA$ where R is the region enclosed by $x - y = 0$, $x - y = 1$, $x + y = 1$,
 $x + y = 3$

Reg No..... Name:.....

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2016
(2015 ADMISSION)

Course Code: MA 101

Course Name: CALCULUS

Max. Marks: 100

Duration: 3 Hours

PART A

(Answer all questions. Each question carries 3 marks)

- 1 Find the derivative of $y = (1 + x \cosh^{-1} x)^2$
- 2 Test the convergence of $\sum_{n=1}^{\infty} \left(\frac{n}{n+1} \right)^{n^2}$
- 3 Classify the surface $4x^2 + 4y^2 + z^2 + 8y - 4z = 4$
- 4 Convert the rectangular co-ordinate into spherical co-ordinate of $(2, 2\sqrt{3}, -4)$
- 5 Prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ where $f = x^2y$.
- 6 Find the velocity, acceleration and speed of a particle moving along the curve
 $x = 1 + 3t, y = 3 - 4t, z = 1 + 3t$ at $t = 2$
- 7 Given $z = e^{xy}, x = 2u + v, y = \frac{u}{v}$ Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$
- 8 Find the unit tangent vector and unit normal vector to the curve
 $x = e^t \cos t, y = e^t \sin t, z = e^t$ at $t = 0$.
- 9 Evaluate $\int_0^3 \int_0^{\sqrt{9-y^2}} 2y dx dy$
- 10 Find the area of the region R enclosed between the parabola $y = \frac{x^2}{2}$ and the
line $y = 2x$

(10*3=30 Marks)

PART B

(Answer any 2 questions each question carries 7 marks)

- 11 Find the radius of curvature and interval of curvature of $\sum_{n=1}^{\alpha} \frac{x^n}{2n+3}$
- 12 Test the convergence of $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \dots$
- 13 Determine the Taylor's series expansion of $f(x) = \sin x$ at $x = \pi/4$.

(Answer any 2 questions each question carries 7 marks)

- 14 Find the nature of domain of the following function
1. $f(x, y) = \sqrt{x^2 - y^2}$
 2. $f(x, y) = \ln(x^2 - y)$
- 15 Show that the function $f(x, y) = \frac{x^3 y}{2x^6 + y^2}$ approaches zero as $(x, y) \rightarrow (0, 0)$ along the line $y = mx$.

- 16 Find the trace of the surface $x^2 + y^2 - z^2 = 0$ in the plane $x = 2$ and $y = 1$.
- $$x^2 + y^2 - z^2 = 0$$

(Answer any 2 questions each question carries 7 marks)

- 17 Find the local linear approximation of $f(x, y) = \sqrt{(x^2 + y^2)}$ at $(3, 4)$ and compare the error in approximation by $L(3.04, 3.98)$ with the distance between the points.
- 18 Find the relative extrema of $f(x, y) = 3x^2 - 2xy + y^2 - 8y$
- 19 If $z = e^{xy}$, $x = 2u + v$, $y = \frac{u}{v}$ Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$

(Answer any 2 questions each question carries 7 marks)

- 20 If $r(t) = e^t i + e^{-2t} j + tk$
- 1) Find the scalar tangential and normal component of acceleration at $t = 0$
 - 2) Find the vector tangential and normal component of acceleration at $t = 0$.
- 21 Find the equation of the tangent plane and parametric equations of the normal

line to the surface $z = 4x^3y^2 + 2y - 2$ at the point $P(1, -2, 10)$.

- 22 Find the directional derivative of $f = x^2y - yz^3 + z$ at $(1, -2, 0)$ in the direction of $\vec{a} = 2\vec{i} + \vec{j} + 2\vec{k}$

(Answer any 2 questions each question carries 7 marks)

- 23 Evaluate $\iint_R y \, dA$ where R is the region in the first quadrant enclosed between the circle $x^2 + y^2 = 25$ and the line $x+y=5$

- 24 Change the order of integration and evaluate $\int_1^{2y^2} \int_y y^2 \, dx \, dy$

- 25 Find the volume bounded by the cylinder $x^2 + y^2 = 4$ the planes $y + z = 3$ and $z = 0$.

Reg. No:.....

Name:.....

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION, FEBRUARY 2017

MA101: CALCULUS

Max. Marks: 100

Duration: 3 Hours

PART A

(Answer All Questions and each carries 5 marks)

1. a) Test the convergence of $\sum_{k=1}^{\infty} \frac{99^k}{k!}$ (2)
- b) Test the convergence of $\sum_{k=1}^{\infty} \frac{1}{3^{k+1}}$. (3)
2. a) Find the slope of the sphere $x^2+y^2+z^2=1$ in the y- direction at $(\frac{2}{3}, \frac{1}{3}, \frac{-2}{3})$. (2)
- b) Find the critical points of the function $f(x,y) = 2xy-x^3-y^3$. (3)
3. a) Find the velocity at time $t=\pi$ of a particle moving along the curve
 $\vec{r}(t) = e^t \sin t \ i + e^t \cos t \ j + t \ k$. (2)
- b) Find the directional derivative of $f(x,y) = xe^y - ye^x$ at the point P(0,0) in the direction of $5i - 2j$. (3)
4. a) Change the order of integration in $\int_0^1 \int_{y^2}^{\sqrt{2-y^2}} f(x,y) dx dy$. (3)
- b) Find the area of the region enclosed by $y=x^2$ and $y=x$. (2)
5. a) Find the divergence of the vector field $f(x,y,z) = x^2y \ i + 2y^3z \ j + 3z \ k$. (2)
- b) Find the work done by $\vec{F} = xy \ i + x^3j$ on a particle that moves along the curve $y^2=x$ from (0,0) to (0,1). (3)
6. a) Using Green's theorem to evaluate $\int_C 2xy \ dx + (x^2 + x) \ dy$ where C is the triangle with vertices (0,0), (1,0) and (1,1). (2)
- b) Use Stoke's theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (x-2y)i + (y-z)j + (z-x)k$ and C is the circle $x^2+y^2 = a^2$ in the xy plane with counter clockwise orientation looking down the positive z- axis. (3)

PART B

MODULE I (Answer Any Two Questions)

7. a) Test the convergence of the following series (5)

$$\text{i) } \sum_{k=1}^{\infty} \frac{(k+4)!}{4!k! 4^k} \quad \text{ii) } \sum_{k=2}^{\infty} \left(\frac{4k-5}{2k+1}\right)^k$$

8. Use the alternating series test to show that the series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{(k+3)}{k(k+1)}$ converge. (5)

9. Find the Taylor's series of $f(x) = x \sin x$ about the point $x = \frac{\pi}{2}$. (5)

MODULE II (Answer Any Two Questions)

10. Find the local linear approximation L to $f(x,y) = \ln(xy)$ at $P(1,2)$ and compare the error in approximating f by L at $Q(1.01, 2.01)$ with the distance between P and Q . (5)

11. Show that the function $f(x,y) = 2 \tan^{-1}(y/x)$ satisfies the Laplace's equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0. \quad (5)$$

12. Find the relative minima of $f(x,y) = 3x^2 - 2xy + y^2 - 8y$. (5)

MODULE III (Answer Any Two Questions)

13. Find the unit tangent vector and unit normal vector to $\vec{r} = 4 \cos t \mathbf{i} + 4 \sin t \mathbf{j} + t \mathbf{k}$ at $t = \frac{\pi}{2}$. (5)

14. Suppose a particle moves through 3- space so that its position vector at time t is

$$\vec{r} = t \mathbf{i} + t^2 \mathbf{j} + t^3 \mathbf{k}. \text{ Find the scalar tangential component of acceleration at the time } t=1. \quad (5)$$

15. Given that the directional derivative of $f(x,y,z)$ at $(3,-2, 1)$ in the direction of $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ is -5 and that $\|\nabla f(3, -2, 1)\| = 5$. Find $\nabla f(3, -2, 1)$. (5)

MODULE IV (Answer Any Two Questions)

16. Evaluate the integral $\int_0^4 \int_{\sqrt{y}}^2 e^{x^3} dx dy$ by reversing the order of integration. (5)

17. Evaluate $\int_0^1 \int_0^{y^2} \int_{-1}^z z dx dy$. (5)

18. Find the volume of the solid in the first octant bounded by the co-ordinate planes and the plane $x+2y+z = 6$. (5)

MODULE V (Answer Any Three Questions)

19. Let $\vec{r} = xi + yj + zk$ and let $r = \|\vec{r}\|$ and f be a differentiable function of one variable
show that $\nabla f(r) = \frac{f'(r)}{r} \vec{r}$. (5)
20. Evaluate the line integral $\int_C [-y dx + x dy]$ along $y^2 = 3x$ from (3,3) to (0,0). (5)
21. Show that $\vec{F}(x,y) = (\cos y + y \cos x) i + (\sin x - x \sin y) j$ is a conservative vector field.
Hence find a potential function for it. (5)
22. Show that the integral $\int_C (3x^2 e^y dx + x^3 e^y dy)$ is independent of the path and hence
evaluate the integral from (0,0) to (3,2). (5)
23. Find the work done by the force field $\vec{F} = xy i + yz j + xz k$ on a particle that moves
along the curve $C: \vec{r}(t) = t i + t^2 j + t^3 k$ where $0 \leq t \leq 1$. (5)

MODULE VI (Answer Any Three Questions)

24. Use Green's theorem to evaluate the integral $\int_C (x \cos y dx - y \sin x dy)$ where
 C is the square with vertices (0,0), (π ,0), (π , π) and (0, π). (5)
25. Evaluate the surface integral $\int_{\sigma} \int z^2 ds$ where σ is the portion of the
cone $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 3$. (5)
26. Use divergence theorem to find the outward flux of the vector field $\vec{F} = 2x i + 3y j + z^2 k$
across the unit cube $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$. (5)
27. Use Stoke's theorem to evaluate the integral $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (x-y) i + (y-z) j + (z-x) k$
and C is the boundary of the portion of the plane $x+y+z = 1$ in the first octant. (5)
28. Use Stoke's theorem to evaluate the integral $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = 2z i + 3x j + 5y k$ and
 C is the boundary of the paraboloid $x^2 + y^2 + z = 4$ for which $z \geq 0$ and C is positively
oriented. (5)

Reg.No.....

Name.....

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITYFIRST SEMESTER B.TECH DEGREE (SUPPLEMENTARY) EXAMINATION,
FEBRUARY 2017 (2015 ADMISSION)Course Code: **MA 101**
Course Name: **CALCULUS**

Max.Marks : 100

Duration : 3 Hours

PART A*(Answer all questions. Each question carries 3 marks)*

- 1) Show that the series $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$ converges.
- 2) Classify the surface $z = (x - 1)^2 + (y + 2)^2 + 3$
- 3) Find the Maclaurin series for $\cos x$
- 4) Evaluate $\lim_{(x,y) \rightarrow (-1,2)} \frac{xy}{x^2 + y^2}$
- 5) Convert the cylindrical co-ordinate into rectangular co ordinate of $(4, \pi/3 - 3)$.
- 6) Find the slope of the surface $z = xy^2$ in the x direction at the point $(2,3)$.
- 7) Find the directional derivative of $f = x^2y - yz^3 + z$ at $(1,-2,0)$ in the direction of
$$\vec{a} = 2\vec{i} + \vec{j} + 2\vec{k}$$
- 8) Find the unit normal to the surface $xy + xz + yz = c$ at $(-1,2,3)$
- 9) Evaluate $\int_1^a \int_1^b x^2y \, dx dy$
- 10) Find the area of the region R enclosed by $y = 1, y = 2, x = 0, x = y$.

PART B*(Answer any 2 questions. Each question carries 7 marks)*

- 11) Test the absolute convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n n^4}{4^n}$
- 12) Determine the Taylor's series expansion of $f(x) = \sin x$ at $x = \pi/2$.
- 13) Test the convergence of $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} \dots\dots\dots$

(Answer any 2 questions. Each question carries 7 marks)

14) Find the equation of the paraboloid $z = x^2 + y^2$ in the cylindrical and spherical coordinates.

15) Find $F(f(x), g(y), h(z))$ if $F(x, y, z) = y e^{xyz}$, $f(x) = x^2$, $g(y) = y + 1$, $h(z) = 2z^2$

16) By converting into polar coordinate evaluate $\lim_{(x,y) \rightarrow (0,0)} \sqrt{x^2 + y^2} \ln \left((x^2 + y^2)^2 \right)$

(Answer any 2 questions. Each question carries 7 marks)

17) Find the local linear approximation L of $f(x, y, z) = xyz$ at the point $P(1, 2, 3)$. Compare the error in approximating f by L at the point $Q(1.001, 2.002, 3.003)$ with the distance PQ .

18) Find the relative extrema of $f(x, y) = 3x^2 - 2xy + y^2 - 8y$

19) If f is a differentiable function of three variables and suppose that

$$w = f(x - y, y - z, z - x) \quad \text{Show that } \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$$

(Answer any 2 questions. Each question carries 7 marks)

20) Suppose that a particle moves along a curve in 3-space so that its position vector at time t is $r(t) = 4\cos \pi t \mathbf{i} + 4\sin \pi t \mathbf{j} + t \mathbf{k}$. Find the distance travelled and the displacement of the particle during the time interval $1 \leq t \leq 5$

21) A particle is moving along the curve, $\vec{r} = (t^3 - 2t)\vec{i} + (t^2 - 4)\vec{j}$ where t denotes the time. Find the scalar tangential and normal components of acceleration at $t = 1$. Also find the vector tangential and normal components of acceleration at $t = 0$.

22) Find the arc length of the parametric curve $x = 5\cos t$, $y = 5\sin t$, $z = 2t$; $0 \leq t \leq \pi$

(Answer any 2 questions. Each question carries 7 marks)

23) Evaluate the integral by converting into polar co ordinates $\int_0^{2\sqrt{4-x^2}} \int_0^x (x^2 + y^2) dy dx$

24) Using triple integral to find the volume bounded by the cylinder

$$x^2 + y^2 = 4 \text{ and the planes } z=0 \text{ and } y+z=3$$

25) Change the order of integration and evaluate $\int_0^1 \int_x^1 \frac{x}{x^2 + y^2} dx dy$

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION, JUNE/JULY 2017

Course Code: MA 101
Course Name: CALCULUS
(For 2015 Admission and 2016 Admission)

Max. Marks :100

Duration: 3 hours

PART A

Answer all questions. Each question carries 5 marks.

1. (a) Find the interval of convergence and radius of convergence of the infinite series $\sum_{n=0}^{\infty} n! x^n$ (2)
 (b) Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{5n-1}$ converges or not (3)
2. (a) Find the slope of the surface $z = \sqrt{3x + 2y}$ in the y-direction at the point (4,2) (2)
 (b) Find the derivative of $w = x^2 + y^2$ with respect to t along the path $x = at^2, y = 2at$ (3)
3. (a) Find the directional derivative of $f(x, y) = xe^y$ at (1,1) in the direction of the vector $i - j$ (2)
 (b) If $\vec{F}(t)$ has a constant direction, then prove that $\vec{F} \times \frac{d\vec{F}}{dt} = 0$ (3)
4. (a) Evaluate $\int_0^1 \int_0^1 \frac{1}{\sqrt{(1-x^2)(1-y^2)}} dx dy$ (2)
 (b) Evaluate $\iint_R \frac{\sin x}{x} dx dy$ where R is the triangular region bounded by the x-axis, $y = x$ and $x = 1$. (3)
5. (a) Show that $\int_A^B (2xy + z^3) dx + x^2 dy + 3xz^2 dz$ is independent of the path joining the points A and B. (2)
 (b) If $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$, then prove that $\nabla^2 r^n = n(n+1)r^{n-2}$ (3)
6. (a) Using line integral evaluate the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (2)
 (b) Evaluate $\int_C (e^x dx + 2y dy - dz)$ where C is the curve $x^2 + y^2 = 4, z = 2$. (3)

PART B

Answer any two questions each Module I to IV

Module I

7. Determine whether the series converge or diverge $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$ (5)
8. Check the absolute convergence or divergence of the series $\sum_{n=1}^{\infty} (-1)^n \frac{(2n-1)!}{3^n}$ (5)

9. Find the Taylor series expansion of $\log \cos x$ about the point $\frac{\pi}{3}$ (5)

Module II

10. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, Show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$ (5)
11. The length, width and height of a rectangular box are measured with an error of at most 5%. Use a total differential to estimate the maximum percentage error that results if these quantities are used to calculate the diagonal of the box. (5)
12. Locate all relative extrema and saddle points of $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$ (5)

Module III

13. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$ (5)
14. Let $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$, then prove that $\nabla f(r) = \frac{f'(r)}{r} \vec{r}$. (5)
15. Find an equation of the tangent plane to the ellipsoid $2x^2 + 3y^2 + z^2 = 9$ at the point $(2, 1, 1)$ and determine the acute angle that this plane makes with the XY plane. (5)

Module IV

16. Change the order of integration and hence evaluate $\int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$ (5)
17. Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} y(x^2 + y^2) \, dx \, dy$ using polar co-ordinates (5)
18. Find the volume of the paraboloid of revolution $x^2 + y^2 = 4z$ cut off by the plane $z = 4$ (5)

Module V

Answer any 3 questions.

19. Evaluate the line integral $\int_C (xy + z^3) \, ds$ from $(1, 0, 0)$ to $(-1, 0, \pi)$ along the helix C that is represented by the parametric equations $x = \cos t, y = \sin t, z = t$ (5)
20. Evaluate the line integral $\int_C (y - x) \, dx + x^2 y \, dy$ along the curve $C, y^2 = x^3$ from $(1, -1)$ to $(1, 1)$ (5)
21. Find the work done by the force field $\vec{F} = (x + y)i + xyj - z^2k$ along the line segment from $(0, 0, 0)$ to $(1, 3, 1)$ and then to $(2, -1, 5)$. (5)
22. Show that $\vec{F} = (2xy + z^3)i + x^2j + 3xz^2k$ is a conservative vector field. Also find its scalar potential. (5)
23. Find the values of constants a, b, c so that $\vec{F} = (axy + bz^3)i + (3x^2 - cz)j + (3xz^2 - y)k$ may be irrotational. For these values of a, b, c find the scalar potential of \vec{F} (5)

Module VI

Answer any 3 questions.

24. Verify Green's theorem for $\int_C (xy + y^2)dx + x^2 dy$ where C is bounded by $y = x$ and $y = x^2$ (5)
25. Apply Green's theorem to evaluate $\int_C (2x^2 - y^2)dx + (x^2 + y^2)dy$ where C is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = a^2$ (5)
26. Apply Stokes theorem to evaluate $\int_C (x + y)dx + (2x - y)dy + (y + z)dz$ where C is the boundary of the triangle with vertices (0,0,0), (2,0,0) and (0,3,0) (5)
27. Use Divergence theorem to evaluate $\iint_S \vec{F} \cdot \vec{n}dS$ where $\vec{F} = xi + zj + yzk$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$. Also verify this result by computing the surface integral over S (5)
28. State Divergence theorem. Also evaluate $\iint_S \vec{F} \cdot \vec{n}dS$ where $\vec{F} = axi + byj + czk$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ (5)

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2017

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks.

- | | | Marks |
|---|-----------------------------------------------------------------------------------------------------------------------------------------------|-------|
| 1 | a) Test the convergence of the series $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{2k-1}}$. | (2) |
| | b) Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{x^n}{2n+3}$. | (3) |
| 2 | a) Find the Slope of the surface $z = xe^{-y} + 5y$ in the y-direction at the point (4,0). | (2) |
| | b) Find the derivative of $z = \sqrt{1+x-2xy^4}$ with respect to t along the path $x = \log t, y = 2t$. | (3) |
| 3 | a) Find the directional derivative of $f = x^2y - yz^3 + z$ at $(-1, 2, 0)$ in the direction of $a = 2i + j + 2k$. | (2) |
| | b) Find the unit tangent vector and unit normal vector to $r(t) = 4 \cos t i + 4 \sin t j + t k$ at $t = \frac{\pi}{2}$. | (3) |
| 4 | a) Evaluate $\int_0^{\log 3} \int_0^{\log 2} e^{x+2y} dy dx$. | (2) |
| | b) Evaluate $\iint_R xy dA$, where R is the region bounded by the curves $y = x^2$ and $x = y^2$. | (3) |
| 5 | (a) Find the divergence and curl of the vector $F(x, y, z) = yz i + xy^2 j + yz^2 k$. | (2) |
| | (b) Evaluate $\int_C (3x^2 + y^2) dx + 2xy dy$ along the circular arc C given by $x = \cos t, y = \sin t$ for $0 \leq t \leq \frac{\pi}{2}$. | (3) |
| 6 | (a) Use line integral to evaluate the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. | (2) |
| | (b) Evaluate $\int_C (x^2 - 3y) dx + 3x dy$, where C is the circle $x^2 + y^2 = 4$. | (3) |

PART B

Module 1

Answer any two questions, each carries 5 marks.

- | | | |
|---|-----------------------------------------------------------------------------------------------------------|-----|
| 7 | Test the convergence or divergence of the series $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$. | (5) |
|---|-----------------------------------------------------------------------------------------------------------|-----|

8 Test the absolute convergence of $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{(2k)!}{(3k-2)!}$. (5)

9 Find the Taylor series for $\frac{1}{1+x}$ at $x=2$. (5)

Module II

Answer any two questions, each carries 5 marks.

10 Find the local linear approximation L to $f(x, y) = \log(xy)$ at P(1,2) and compare the error in approximating f by L at Q(1.01, 2.01) with the distance between P and Q. (5)

11 Let $w = 4x^2 + 4y^2 + z^2$, $x = \rho \sin \phi \cos \theta$, $y = \rho \sin \phi \sin \theta$, $z = \rho \cos \phi$. Find $\frac{\partial w}{\partial \rho}$, $\frac{\partial w}{\partial \phi}$ and $\frac{\partial w}{\partial \theta}$. (5)

12 Locate all relative extrema and saddle points of $f(x, y) = 4xy - x^4 - y^4$. (5)

Module III

Answer any two questions, each carries 5 marks.

13 Find the equation of the tangent plane and parametric equation for the normal line to the surface $x^2 + y^2 + z^2 = 25$ at the point (3,0,4). (5)

14 A particle is moving along the curve $r(t) = (t^3 - 2t)i + (t^2 - 4)j$ where t denotes the time. Find the scalar tangential and normal components of acceleration at $t=1$. Also find the vector tangential and normal components of acceleration at $t=1$. (5)

15 The graphs of $r_1(t) = t^2i + tj + 3t^3k$ and $r_2(t) = (t-1)i + \frac{1}{4}t^2j + (5-t)k$ are intersect at the point P(1,1,3). Find, to the nearest degree, the acute angle between the tangent lines to the graphs of $r_1(t)$ & $r_2(t)$ at the point P(1,1,3). (5)

Module IV

Answer any two questions, each carries 5 marks.

16 Change the order of integration and evaluate $\int_0^1 \int_{4x}^4 e^{-y^2} dy dx$. (5)

17 Use triple integral to find the volume bounded by the cylinder $x^2 + y^2 = 9$ and between the planes $z=1$ and $x+z=5$. (5)

18 Find the area of the region enclosed between the parabola $y = \frac{x^2}{2}$ and the line $y = 2x$. (5)

Module V

Answer any three questions, each carries 5 marks.

19 Determine whether $F(x, y) = (\cos y + y \cos x)i + (\sin x - x \sin y)j$ is a conservative vector field. If so find the potential function for it. (5)

20 Show that the integral $\int_{(1,1)}^{(3,3)} (e^x \log y - \frac{e^y}{x}) dx + (\frac{e^x}{y} - e^y \log x) dy$, where x and y are positive is independent of the path and find its value. (5)

21 Find the work done by the force field $F(x, y, z) = xyi + yzj + xzk$ on a particle that moves along the curve $C : r(t) = ti + t^2j + t^3k$ ($0 \leq t \leq 1$). (5)

22 Let $\vec{r} = xi + yj + zk$ and $r = \|\vec{r}\|$, let f be a differentiable function of one variable, (5)
then show that $\nabla f(r) = \frac{f'(r)}{r} \vec{r}$.

23 Find $\nabla \cdot (\nabla \times F)$ and $\nabla \times (\nabla \times F)$ where $F(x, y, z) = e^{xz}i + 4xe^y j - e^{yz}k$. (5)

Module VI

Answer any three questions, each carries 5 marks.

24 Use Green's Theorem to evaluate $\int_C \log(1+y)dx - \frac{xy}{(1+y)}dy$, where C is the (5)
triangle with vertices $(0,0)$, $(2,0)$ and $(0,4)$.

25 Evaluate the surface integral $\iint_{\sigma} xz ds$, where σ is the part of the plane $x + y + z = 1$ (5)
that lies in the first octant.

26 Using Stoke's Theorem evaluate $\int_C F \cdot dr$ where $F(x, y, z) = xzi + 4x^2y^2j + yxk$, C (5)
is the rectangle $0 \leq x \leq 1, 0 \leq y \leq 3$ in the plane $z = y$.

27 Using Divergence Theorem evaluate $\iint_{\sigma} \vec{F} \cdot n ds$ where (5)
 $F(x, y, z) = x^3i + y^3j + z^3k$, σ is the surface of the cylindrical solid bounded by
 $x^2 + y^2 = 4, z = 0$ and $z = 4$.

28 Determine whether the vector fields are free of sources and sinks. If it is not, (5)
locate them
(i) $(y+z)i - xz^3j + x^2 \sin yk$ (ii) $xyi - 2xyj + y^2k$

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks.

- | | | Marks |
|---|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| 1 | a) Determine whether the series $\sum_{k=0}^{\infty} \frac{5}{4^k}$ converges. If so, find the sum | (2) |
| | b) Examine the convergence of $\sum \left(\frac{k}{k+1}\right)^{k^2}$ | (3) |
| 2 | a) Find the slope of the surface $z = x e^{-y} + 5y$ in the y direction at the point (4, 0) | (2) |
| | b) Show the function $f(x, y) = e^x \sin y + e^y \cos x$ satisfies the Laplace's equation $f_{xx} + f_{yy} = 0$ | (3) |
| 3 | a) Find the directional derivative of $f(x, y, z) = x^3 z - yx^2 + z^2$ at P (2, -1, 1) in the direction of $3\vec{i} - \vec{j} + 2\vec{k}$ | (2) |
| | b) Find the unit tangent vector and unit normal vector to the curve $\mathbf{r}(t) = 4\cos t \mathbf{i} + 4 \sin t \mathbf{j} + t \mathbf{k}$ at $t = \frac{\pi}{2}$ | (3) |
| 4 | a) Using double integration, evaluate the area enclosed by the lines $x = 0, y = 0, \frac{x}{a} + \frac{y}{b} = 1$ | (2) |
| | b) Evaluate $\int_{-1}^2 \int_0^2 \int_0^1 (x^2 + y^2 + z^2) dx dy dz$ | (3) |
| 5 | a) If $\mathbf{F}(x, y, z) = x^2 \mathbf{i} - 3\mathbf{j} + yz^2 \mathbf{k}$ find $\text{div } \mathbf{F}$ | (2) |
| | b) Find the work done by the force field $\mathbf{F} = xy \mathbf{i} + yz \mathbf{j} + zx \mathbf{k}$ on a particle that moves along the curve C: $x = t, y = t^2, z = t^3, 0 \leq t \leq 1$ | (3) |
| 6 | a) Use Green's theorem to evaluate $\int_c (x dy - y dx)$, where c is the circle $x^2 + y^2 = a^2$ | (2) |
| | b) If S is any closed surface enclosing a volume V and $\mathbf{F} = x\mathbf{i} + 2y\mathbf{j} + 3z\mathbf{k}$ show that $\iint_S \mathbf{F} \cdot \mathbf{n} ds = 6V$ | (3) |

PART B

Module I

Answer any two questions, each carries 5 marks.

- | | | |
|---|-----------------------------------------------------------------------------------------------------------------------|-----|
| 7 | Determine whether the alternating series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+7}{k(k+4)}$ is absolutely convergent | (5) |
| 8 | Find the Taylor series expansion of $f(x) = \frac{1}{x+2}$ about $x = 1$ | (5) |
| 9 | Find the interval of convergence and radius of convergence of $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{(x+1)^k}{k}$ | (5) |

Module II

Answer any two questions, each carries 5 marks.

- | | | |
|----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 10 | Find the local linear approximation L to the function $f(x, y, z) = xyz$ at the point P (1,2,3). Also compare the error in approximating f by L at the point Q (1.001, 2.002, 3.003) with the distance PQ. | (5) |
| 11 | Locate all relative extrema and saddle points of $f(x, y) = 2xy - x^3 - y^2$ | (5) |
| 12 | If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$ | (5) |

Module III

Answer any two questions, each carries 5 marks.

- 13 Write the parametric equations of the tangent line to the graph of $\mathbf{r}(t) = \ln t \mathbf{i} + e^{-t} \mathbf{j} + t^4 \mathbf{k}$ at $t = 2$ (5)
- 14 A particle moves along the curve $\mathbf{r} = (t^3 - 4t) \mathbf{i} + (t^2 + 4t) \mathbf{j} + (8t^2 - 3t^3) \mathbf{k}$ where t denotes time. Find (5)
- (i) the scalar tangential and normal components of acceleration at time $t = 2$
- (ii) the vector tangential and normal components of acceleration at time $t = 2$
- 15 Find the equation to the tangent plane and parametric equations of the normal line to the ellipsoid $x^2 + y^2 + 4z^2 = 12$ at the point $(2, 2, 1)$ (5)

Module IV

Answer any two questions, each carries 5 marks.

- 16 Reverse the order of integration and evaluate $\int_0^1 \int_x^1 \frac{x}{x^2 + y^2} dy dx$ (5)
- 17 If R is the region bounded by the parabolas $y = x^2$ and $y^2 = x$ in the first quadrant, evaluate $\iint_R (x + y) dA$ (5)
- 18 Use triple integral to find the volume of the solid bounded by the surface $y = x^2$ and the planes $y + z = 4, z = 0$. (5)

Module V

Answer any three questions, each carries 5 marks.

- 19 If $\mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$ and $r = \|\mathbf{r}\|$, show that $\nabla \log r = \frac{\mathbf{r}}{r^2}$ (5)
- 20 Examine whether $\mathbf{F} = (x^2 - yz) \mathbf{i} + (y^2 - zx) \mathbf{j} + (z^2 - xy) \mathbf{k}$ is a conservative field. If so, find the potential function (5)
- 21 Show that $\nabla^2 f(r) = 2 \frac{f'(r)}{r} + f''(r)$, where $\mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}, r = \|\mathbf{r}\|$ (5)
- 22 Compute the line integral $\int_C (y^2 dx - x^2 dy)$ along the triangle whose vertices are $(1, 0), (0, 1)$ and $(-1, 0)$ (5)
- 23 Show that the line integral $\int_C (y \sin x dx - \cos x dy)$ is independent of the path and hence evaluate it from $(0, 1)$ and $(\pi, -1)$ (5)

Module VI

Answer any three questions, each carries 5 marks.

- 24 Using Green's theorem, find the work done by the force field $\vec{f}(x, y) = (e^x - y^3) \vec{i} + (\cos y + x^3) \vec{j}$ on a particle that travels once around the unit circle $x^2 + y^2 = 1$ in the counter clockwise direction. (5)
- 25 Using Green's theorem evaluate $\int_C (xy + y^2) dx + x^2 dy$, where C is the boundary of the area common to the curve $y = x^2$ and $y = x$ (5)
- 26 Evaluate the surface integral $\iint_S xz ds$, where S is the part of the plane $x + y + z = 1$ that lies in the first octant (5)
- 27 Using divergence theorem, evaluate $\iint_S \mathbf{F} \cdot \mathbf{n} ds$ where $\mathbf{F} = (x^2 + y) \mathbf{i} + z^2 \mathbf{j} + (e^y - z) \mathbf{k}$ and S is the surface of the rectangular solid bounded by the coordinate planes and the planes $x = 3, y = 1, z = 3$ (5)
- 28 Apply Stokes's theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = (x^2 - y^2) \mathbf{i} + 2xy \mathbf{j}$ and C is the rectangle in the xy plane bounded by the lines $x = 0, y = 0, x = a$ and $y = b$ (5)

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION, JULY 2018

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks

- | | | Marks |
|---|----------------------------------------------------------------------------------------------------------------------|-------|
| 1 | a) Express the repeating decimal 1.454545... as a fraction. | (2) |
| | b) Expand $f(x) = \sin \pi x$ using Taylor series about $x = \frac{1}{2}$. | (3) |
| 2 | a) Find the slope of the surface $z = \sqrt{3x + 2y}$ in the x-direction at the point (1,3). | (2) |
| | b) Find a linear approximation of $f(x, y, z) = x^2 - xy + 3 \sin z$ at the point (2,1,0). | (3) |
| 3 | a) Determine whether $\vec{r}(t) = 3 \sin t \hat{i} + 3t \hat{j}$ is continuous at $t = 0$. | (2) |
| | b) Find the gradient of $f(x, y) = x^2 e^y$ at (1,0). | (3) |
| 4 | a) Evaluate $\int_0^1 \int_0^{\log y} \frac{dx dy}{\log y}$. | (2) |
| | b) Using double integral find the area bounded by $x = y^2, x + y = 2$ in the positive quadrant. | (3) |
| 5 | a) Find the divergence of vector field $\vec{V} = x^2 y^2 \hat{i} + 2xy \hat{j}$. | (2) |
| | b) Show that the vector field $\vec{F} = 2x(y^2 + z^3) \hat{i} + 2x^2 y \hat{j} + 3x^2 z^2 \hat{k}$ is conservative. | (3) |
| 6 | a) Locate sources or sinks of $\vec{F} = xy \hat{i} - 2xy \hat{j} + y^2 \hat{k}$. | (2) |
| | b) By Green's theorem evaluate $\oint_C (3x + y) dx + (2x + y) dy$, C is the circle $x^2 + y^2 = a^2$. | (3) |

PART B

Module I

Answer any two questions, each carries 5 marks

- | | | |
|---|------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 7 | Determine whether $\sum_{k=1}^{\infty} \frac{1}{(3k-1)(3k+2)}$ is convergent. | (5) |
| 8 | Test the convergence of $1 + \frac{1.3}{3!} + \frac{1.3.5}{5!} + \frac{1.3.5.7}{7!} + \dots$ | (5) |
| 9 | Expand $f(x) = \frac{1}{x}$ in powers of (x-2). Find the interval of convergence and radius of convergence of the power series obtained. | (5) |

Module II*Answer any two questions, each carries 5 marks*

- 10 If $W = f(x - y, y - z, z - x)$, Show that $\frac{\partial W}{\partial x} + \frac{\partial W}{\partial y} + \frac{\partial W}{\partial z} = 0$. (5)
- 11 a) If $f(x, y) = y \sin x + e^{2x} \cos y$, find f_{xxx} . (3)
- b) State the second partial test for local (relative) extreme values. (2)
- 12 An aquarium with rectangular sides and bottom (no top) is to hold 32 liters of water. Find its dimension so that least quantity of material is required for its construction. (5)

Module III*Answer any two questions, each carries 5 marks*

- 13 Find the unit tangent $\hat{T}(t)$ and unit normal $\hat{N}(t)$ to the curve (5)
 $\vec{r}(t) = e^t \cos t \hat{i} + e^t \sin t \hat{j} + e^t \hat{k}$ at $t=0$.
- 14 Suppose the position vector of a moving particle is $\vec{r} = t^2 \hat{i} + \frac{1}{3} t^3 \hat{j}$, find the (5)
displacement and distance traveled over the time interval $2 \leq t \leq 4$.
- 15 Find the equation of tangent plane to the surface $x^2 + y^2 + z^2 = 25$ at (5)
 $P(3, 0, 4)$. Also find the parametric equation for the normal line to the surface at P.

Module IV*Answer any two questions, each carries 5 marks*

- 16 Evaluate $\iint_R e^{x^2} dx dy$, where the region R is given by $2y \leq x \leq 2$ and $0 \leq y \leq 1$. (5)
- 17 Evaluate $\iint r \sin \theta dr d\theta$ over the cardioid $r = a(1 - \cos \theta)$ above the initial line. (5)
- 18 Evaluate the triple integral $\iiint xy \sin(yz) dV$ over the rectangular box (5)
 $0 \leq x \leq \frac{\pi}{2}, 0 \leq y \leq 1, 0 \leq z \leq \frac{\pi}{6}$.

Module V*Answer any three questions, each carries 5 marks*

- 19 Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ where $r = \sqrt{x^2 + y^2 + z^2}$. (5)
- 20 Evaluate $\int_C y dx + z dy + x dz$ along the twisted cubic $x = t, y = t^2, z = t^3$ from (5)
 $(0, 0, 0)$ to $(1, 1, 1)$.
- 21 Find the potential function for the vector field (5)
 $\vec{F} = (\sin z + y \cos x) \hat{i} + (\sin x + z \cos y) \hat{j} + (\sin y + x \cos z) \hat{k}$.
- 22 Find the work done in moving a particle in the force field (5)
 $\vec{F} = (x + y) \hat{i} - x^2 \hat{j} + (y + z) \hat{k}$ along the curve defined by $x^2 = 4y, z = x, 0 \leq x \leq 2$.
- 23 Show that $\int_C (yz - 1) dx + (z + xz + z^2) dy + (y + xy + 2yz) dz$ is independent of the (5)
path of integration. Find the scalar potential and the value of the integral from $(1, 2, 2)$ to $(2, 3, 4)$.

Module VI*Answer any three questions, each carries 5 marks*

- 24 Use Green's theorem to evaluate $\int_C x^2 y dx + x dy$ where C is the boundary of (5)
triangular region enclosed by $y = 0, y = 2x, x = 1$.
- 25 Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ using Green's theorem. (5)
- 26 Evaluate the surface integral $\iint_{\sigma} xy dS$ where σ is the portion of the plane (5)
 $x + y + z = 2$ lying in the first octant.
- 27 Using divergence theorem evaluate $\iint_S \vec{F} \cdot \hat{n} dS$ where $\vec{F} = 2x\hat{i} + 4y\hat{j} - 3z\hat{k}$ and S is (5)
the surface of the sphere $x^2 + y^2 + z^2 = 1$.
- 28 Use Stokes' theorem to calculate the circulation around the bounding circle (5)
 $x^2 + y^2 = 9$ and the field $\vec{A} = y\hat{i} - x\hat{j}$ where S is the disk of radius 3 centered at
origin in the XY-plane.

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2018

Course Code: MA101
Course Name: CALCULUS

Max. Marks: 100

Duration: 3 Hours

PART A*Answer all questions, each carries 5 marks.*

Marks

- 1 a) Test the convergence of $\sum_{k=1}^{\infty} \frac{\cos k}{k^2}$ (2)
- b) Discuss the convergence of $\sum_{k=1}^{\infty} \frac{(2k)!}{4^k}$ (3)
- 2 a) Find the slope of the surface $z = \sin(y^2 - 4x)$ in the x - direction at the point $(3, 1)$. (2)
- b) Find the differential dz of the function $z = \tan^{-1}(x^2y)$. (3)
- 3 a) Find the direction in which the function $f(x, y) = xe^{xy}$ decreases fastest at the point $(2, 0)$. (2)
- b) Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at $(1, 1, 3)$ (3)
- 4 a) Evaluate $\iint_R y \sin xy \, dA$, where $R = [1, 2] \times [0, \pi]$. (2)
- b) Evaluate $\int_0^2 \int_0^1 \frac{x}{(1+xy)^2} \, dy \, dx$ (3)
- 5 a) if $\vec{A} = (3x^2 + 6y)\mathbf{i} - 14yz\mathbf{j} + 20xz^2\mathbf{k}$, evaluate $\int \vec{A} \cdot d\vec{r}$ from $(0, 0, 0)$ to $(1, 1, 1)$ (2)
along the path, $x = t, y = t^2, z = t^3$
- b) Prove that $\vec{F} = (x^2 - yz)\mathbf{i} + (y^2 - xz)\mathbf{j} + (z^2 - xy)\mathbf{k}$ is irrotational. (3)
- 6 a) Determine the source and sink of the vector field (2)
 $F(x, y, z) = 2(x^3 - 2x)\mathbf{i} + 2(y^3 - 2y)\mathbf{j} + 2(z^3 - 2z)\mathbf{k}$
- b) Evaluate $\iint_S \vec{F} \cdot \vec{n} \, ds$ where S is the surface of the cylinder $x^2 + y^2 = 4, z = 0,$ (3)
 $z = 3$ where $\vec{F} = (2x - y)\mathbf{i} + (2y - z)\mathbf{j} + z^2\mathbf{k}$

PART B**Module 1***Answer any two questions, each carries 5 marks.*

- 7 Check the convergence of the series $\frac{3}{4} + \frac{3.4}{4.6} + \frac{3.4.5}{4.6.8} + \frac{3.4.5.6}{4.6.8.10} + \dots$ (5)
- 8 Find the radius of convergence of the power series $\sum_{k=1}^{\infty} \frac{(-1)^k (x-4)^k}{3^k}$ (5)
- 9 Determine whether the alternating series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{3^{2k-1}}{k^2+1}$ is absolutely (5)

convergent.

Module II

Answer any two questions, each carries 5 marks.

10 If $u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, find $\frac{\partial^2 u}{\partial x \partial y}$ (5)

11 Let $z = xye^{\frac{x}{y}}$, $x = r \cos \theta$, $y = r \sin \theta$, use chain rule to evaluate $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$ at $r = 2$ and $\theta = \frac{\pi}{6}$ (5)

12 A rectangular box open at the top is to have volume $32m^3$. Find the dimensions of the box requiring least material for its construction. (5)

Module III

Answer any two questions, each carries 5 marks.

13 Suppose that a particle moves along a circular helix in 3-space so that its position vector at time t is $\mathbf{r}(t) = 4\cos \pi t \mathbf{i} + 4\sin \pi t \mathbf{j} + t \mathbf{k}$. Find the distance travelled and the displacement of the particle during the time interval $1 \leq t \leq 5$. (5)

14 Suppose that the position vector of a particle moving in a plane $\bar{r} = 12\sqrt{t} \mathbf{i} + t^{\frac{3}{2}} \mathbf{j}$, $t > 0$. Find the minimum speed of the particle and locate when it has minimum speed? (5)

15 Find the parametric equation of the tangent line to the curve $x = \cos t$, $y = \sin t$, $z = t$ where $t = t_0$ and use this result to find the parametric equation of the tangent line to the point where $t = \pi$. (5)

Module IV

Answer any two questions, each carries 5 marks.

16 Evaluate $\iint_R y \, dA$ where R is the region in the first quadrant enclosed between the circle $x^2 + y^2 = 25$ and the line $x + y = 5$. (5)

17 Evaluate $\int_1^2 \int_0^x \frac{dy \, dx}{x^2 + y^2}$ (5)

18 Evaluate $\iiint_V x \, dx \, dy \, dz$ where V is the volume of the tetrahedron bounded by the plane $x = 0, y = 0, z = 0$ $x + y + z = a$. (5)

Module V

Answer any three questions, each carries 5 marks.

19 Find the scalar potential of $\vec{F} = (2xy + z^3)\mathbf{i} + x^2\mathbf{j} + 3xz^2\mathbf{k}$ (5)

20 Find the work done by $F(x, y) = (x^2 + y^2)\mathbf{i} - x\mathbf{j}$ along the curve $C: x^2 + y^2 = 1$ counter clockwise from $(1,0)$ to $(0,1)$. (5)

- 21 Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = y^2\vec{i} + xy\vec{j}$ and $\vec{r}(t) = t\vec{i} + 2t\vec{j}$, $1 \leq t \leq 3$. (5)
- 22 Evaluate $\int y dx + z dy + x dz$ along the path $x = \cos \pi t, y = \sin \pi t, z = t$ from $(1,0,0)$ to $(-1,0,1)$ (5)
- 23 If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $\rho = \|\vec{r}\|$, prove that $\nabla^2 f(\rho) = \frac{2}{\rho} f'(\rho) + f''(\rho)$. (5)

Module VI

Answer any three questions, each carries 5 marks.

- 24 Using Stoke's theorem evaluate $\int_C \vec{F} \cdot d\vec{r}$; where $\vec{F} = xy\vec{i} + yz\vec{j} + xz\vec{k}$; C triangular path in the plane $x + y + z = 1$ with vertices at $(1,0,0), (0,1,0)$ and $(0,0,1)$ in the first octant (5)
- 25 Using Green's theorem evaluate $\int_C (y^2 - 7y)dx + (2xy + 2x)dy$ where C is the circle $x^2 + y^2 = 1$ (5)
- 26 Find the mass of the lamina that is the portion of the cone $z = \sqrt{x^2 + y^2}$ between $z = 1$ and $z = 3$ if the density is $\rho(x, y, z) = x^2z$. (5)
- 27 Use divergence theorem to find the outward flux of the vector field $F(x, y, z) = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$ across the surface σ bounded by $x^2 + y^2 = 4, z = 0$ and $z = 4$. (5)
- 28 If S is the surface of the sphere $x^2 + y^2 + z^2 = 1$, Evaluate $\iint_S (xi + 2yj + 3zk) \cdot dS$ (5)

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION (S), MAY 2019

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks.

Marks

- 1 a) Check the convergence of the series $\sum_{k=1}^{\infty} \left(\frac{3k-4}{4k-5}\right)^k$ (2)
- b) Find the Maclaurin series of $f(x) = \frac{1}{1+x}$, up to 3 terms (3)
- 2 a) If $z = (3x - 2y)^4$, find $\frac{\partial^4 z}{\partial x \partial y^3}$ (2)
- b) If $w = \log(\tan x + \tan y + \tan z)$ then prove that (3)
- $$\sin 2x \frac{\partial w}{\partial x} + \sin 2y \frac{\partial w}{\partial y} + \sin 2z \frac{\partial w}{\partial z} = 2$$
- 3 a) Find the speed of a particle moving along the path $x = 2 \cos t, y = 2 \sin t, z = t$ (2)
- at $t = \pi/2$
- b) If $y'(t) = \cos t \, i + \sin t \, j$; $y(0) = i - j$. Find $y(t)$. (3)
- 4 a) Evaluate $\int_0^1 \int_0^{x^2} \int_0^2 dy \, dz \, dx$ (2)
- b) Evaluate $\iint xy \, dx \, dy$ over the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and lying (3)
- in the first quadrant.
- 5 a) Show that $F(x, y) = 2xy^3i + 3x^2y^2j$ is conservative. (2)
- b) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = \|\vec{r}\|$, prove that $\nabla \cdot \frac{\vec{r}}{r^3} = 0$ (3)
- 6 a) Evaluate by Stoke's theorem $\oint_C (e^x \, dx + 2y \, dy - dz)$, where C is the curve (2)
- $$x^2 + y^2 = 4, z = 2$$
- b) Using Green's theorem evaluate $\int_C x \, dy - y \, dx$ where C is the circle $x^2 + y^2 = 4$ (3)

PART B

Module 1

Answer any two questions, each carries 5 marks.

- 7 Test for convergence of the series $\sum_{k=1}^{\infty} \frac{1}{(8k^2 - 3k)^{1/3}}$ (5)
- 8 Find the radius of convergence and interval of convergence of the power (5)
- series $\sum_{k=0}^{\infty} \frac{(2x-1)^k}{3^{2k}}$.

- 9 Show that the series $\sum_{k=1}^{\infty} (-1)^k \left(\frac{k}{k+1}\right)^{k^2}$ is convergent. (5)

Module 1I

Answer any two questions, each carries 5 marks.

- 10 Let $w = \sqrt{x^2 + y^2 + z^2}$, where $x = \cos \theta$, $y = \sin \theta$, $z = \tan \theta$. Find $\frac{dw}{d\theta}$ at $\theta = \frac{\pi}{4}$, using chain rule. (5)

- 11 Find the local linear approximation $L(x, y)$ to $f(x, y) = \ln(xy)$ at the point $P(1, 2)$. Compare the error in approximating f by L at the point $Q(1.01, 2.01)$ with the distance between P and Q . (5)

- 12 Find relative extrema and saddle points, if any, of the function (5)
 $f(x, y) = xy + \frac{8}{x} + \frac{8}{y}$.

Module 1II

Answer any two questions, each carries 5 marks.

- 13 Find the unit tangent $T(t)$ and unit normal $N(t)$ to the curve (5)
 $x = a \cos t$, $y = a \sin t$, $z = ct$ $a > 0$

- 14 Find the velocity and position vectors of the particle, if the acceleration vector (5)
 $\mathbf{a}(t) = \sin t \mathbf{i} + \cos t \mathbf{j} + e^t \mathbf{k}$; $\mathbf{v}(0) = \mathbf{k}$; $\mathbf{r}(0) = -\mathbf{i} + \mathbf{k}$.

- 15 Find the equation of the tangent line to the curve of intersection of surfaces (5)
 $z = x^2 + y^2$ and $3x^2 + 2y^2 + z^2 = 9$ and the point $(1, 1, 2)$.

Module 1V

Answer any two questions, each carries 5 marks.

- 16 Evaluate by reversing the order of integration (5)
 $\int_0^{a/\sqrt{2}} \int_y^{\sqrt{a^2-y^2}} x \, dx \, dy$

- 17 Evaluate $\iint_R xy \, dA$, where R is the sector in the first quadrant bounded by (5)

$$y = \sqrt{x}, \quad y = 6 - x, \quad y = 0.$$

- 18 Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dz \, dx \, dy$ (5)

Module V

Answer any three questions, each carries 5 marks.

- 19 Find the work done by $F(x, y) = (x^2 + y^2)i - xj$ along the curve $C: x^2 + y^2 = 1$ counter clockwise from $(1,0)$ to $(0,1)$ (5)
- 20 Determine whether $F(x, y) = 6y^2 i + 12xy j$ is a conservative vector field. If so find the potential function for it. (5)
- 21 Find the divergence and curl of the vector field $F(x, y, z) = xyz^2 i + yzx^2 j + zxy^2 k$ (5)
- 22 Prove that $\int_C (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k} \cdot d\bar{r}$ is independent of the path and evaluate the integral along any curve from $(0,0,0)$ to $(1,2,3)$. (5)
- 23 If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ and $r = \|\bar{r}\|$, prove that $\nabla^2 f(r) = \frac{2}{r} f'(r) + f''(r)$. (5)

Module VI

Answer any three questions, each carries 5 marks.

- 24 Using Green's theorem evaluate $\int_C (xy + y^2)dx + x^2 dy$ where C is the boundary of the region bounded by $y = x^2$ and $x = y^2$ (5)
- 25 Evaluate the surface integral $\iint_{\sigma} z^2 ds$, where σ is the portion of the curve $z = \sqrt{x^2 + y^2}$ between $z = 1$ and $z = 3$ (5)
- 26 Determine whether the vector field $F(x, y, z)$ is free of sources and sinks. If not, locate them. (5)
- i) $F(x, y, z) = (y + z)\bar{i} - xz\bar{j} + x^2 \sin y \bar{k}$
ii) $F(x, y, z) = x^3\bar{i} + y^3\bar{j} + 2z^3\bar{k}$
- 27 Use divergence theorem to find the outward flux of the vector field $F(x, y, z) = (2x + y^2)i + xy j + (xy - 2z)k$ across the surface σ of the tetrahedron bounded by $x + y + z = 2$ and the coordinate planes. (5)
- 28 Using Stoke's theorem evaluate $\int_C \bar{F} \cdot d\bar{r}$; where $\bar{F} = xy\bar{i} + yz\bar{j} + xz\bar{k}$; C triangular path in the plane $x + y + z = 1$ with vertices at $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$ in the first octant (5)

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech examinations (S) September 2020 S1/S2 (2015 Scheme)

Course Code: MA101**Course Name: CALCULUS**

Max. Marks: 100

Duration: 3 Hours

PART A*Answer all questions, each carries 5 marks.*

Marks

- 1 a) Determine whether the series $\sum_{k=1}^{\infty} \left(\frac{2}{3}\right)^{k+2}$ converges. If so, find the sum (2)
- b) Find the Maclaurin series expansion of $f(x) = \ln(1-x)$ up to 3 terms (3)
- 2 a) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $ye^x - 5\cos 2z = 3z$ (2)
- b) Use chain rule to find $\frac{dw}{dx}$ at $(0,1,2)$ for $w = xy + yz$, $y = \sin x$, $z = e^x$. (3)
- 3 a) Find the velocity of a particle moving along the curve (2)
- $$\vec{r}(t) = e^t \sin t \vec{i} + e^t \cos t \vec{j} + t \vec{k} \quad \text{at } t = \pi$$
- b) Find the unit normal to the surface $yz + zx + xy = c$ at $(-1,2,3)$ (3)
- 4 a) Evaluate $\int_1^2 \int_y^{3-y} dx dy$ (2)
- b) Evaluate $\int_1^2 \int_0^x \frac{dy dx}{x^2 + y^2}$. (3)
- 5 a) Find the value of constant a so that if (2)
- $$\vec{F} = (3x - 2y + z) \vec{i} + (4x - ay + z) \vec{j} + (x - y + 2z) \vec{k}$$
- is solenoidal.
- b) Find the work done by a force field $F(x,y) = -y\vec{i} + x\vec{j}$ acting on a particle (3)
- moving along the circle $x^2 + y^2 = 3$ from $(\sqrt{3}, 0)$ to $(0, \sqrt{3})$
- 6 a) Determine the source and sink of the vector field $F(x,y,z) = 2(x^3 - 2x)\vec{i} +$ (2)
- $$2(y^3 - 2y)\vec{j} + 2(z^3 - 2z)\vec{k}$$
- b) Using Stoke's theorem prove that $\int_C \vec{r} \cdot d\vec{r} = 0$ where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and (3)
- C is any closed curve.

PART B**Module 1***Answer any two questions, each carries 5 marks.*

7 Test the convergence of the infinite series $\sum_{k=1}^{\infty} \left(\frac{k}{k+1}\right)^{k^2}$. (5)

8 Examine the convergence of $\sum_{k=0}^{\infty} \frac{(k+4)!}{4!k!4^k}$ (5)

9 Find the radius of convergence of the power series $\sum_{k=1}^{\infty} \frac{(2x-3)^k}{4^{2k}}$ (5)

Module 1I*Answer any two questions, each carries 5 marks.*

10 The height and radius of a circular cone is measured with errors of atmost 3% and 5% respectively. Use differentials to approximate the maximum percentage error in calculated volume. (5)

11 If $u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, find $\frac{\partial^2 u}{\partial x \partial y}$ (5)

12 Find relative extrema and saddle points, if any, of the function $f(x, y) = x^3 + y^3 - 15xy$. (5)

Module 1II*Answer any two questions, each carries 5 marks.*

13 Find where the tangent line to the curve $\mathbf{r}(t) = e^{-2t}\mathbf{i} + \cos t \mathbf{j} + 3 \sin t \mathbf{k}$ at the point (1,1,0) intersects the YZ plane. (5)

14 Find the position and velocity vectors of the particle given $\mathbf{a}(t) = (t+1)^{-2}\mathbf{j} - e^{-2t}\mathbf{k}$, $\mathbf{v}(0) = 3\mathbf{i} - \mathbf{j}$, $\mathbf{r}(0) = \mathbf{k}$ (5)

15 A particle moves along a curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$ where t is the time. Find the component of acceleration at time $t = 1$ in the direction of $\vec{i} - 3\vec{j} + 2\vec{k}$ (5)

Module 1V*Answer any two questions, each carries 5 marks.*

16 Evaluate $\iiint_R xy \sin z \, dV$ where R is the rectangular box defined by $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq \frac{\pi}{6}$ (5)

17 Sketch the region of integration and evaluate $\int_1^2 \int_y^{y^2} dx \, dy$ by changing the order of integration. (5)

18 Use double integral to find the area bounded by the x – axis $y = 2x$ and $x + y = 1$ (5)

Module V*Answer any three questions, each carries 5 marks.*

- 19 Prove that $\int_C (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k} \cdot d\bar{r}$ is independent of the path and evaluate the integral along any curve from (0,0,0) to (1,2,3). (5)
- 20 Evaluate $\int_C xy^2 dx + xy dy$ where C is a triangle with vertices at (0,0), (0,1) and (2,1) (5)
- 21 Evaluate $\int_C 2xy dx + (x^2 + y^2)dy$ along the curve $C: x = \cos t, y = \sin t, 0 \leq t \leq \frac{\pi}{2}$ (5)
- 22 Determine whether $F(x, y) = 6y^2 i + 12xy j$ is a conservative vector field. If so find the potential function for it. (5)
- 23 If $\bar{F} = (\sin z + y \cos x)\bar{i} + (\sin x + 2 \cos y)\bar{j} + (\sin y + x \cos z)\bar{k}$, find $Div \bar{F}$ and $Curl \bar{F}$. (5)

Module VI*Answer any three questions, each carries 5 marks.*

- 24 Using Stoke's theorem, evaluate $\int_C \bar{F} \cdot d\bar{r}$ where C is the boundary of the projection of the sphere $x^2 + y^2 + z^2 = 1$ on the XY plane with $\bar{F} = (2x - y)\bar{i} - yz^2\bar{j} - y^2z\bar{k}$ (5)
- 25 Using Green's theorem evaluate $\int_C (y^2 - 7y)dx + (2xy + 2x) dy$ where C is the circle $x^2 + y^2 = 1$ (5)
- 26 Evaluate using divergence theorem for $\bar{F} = x^2\bar{i} + z\bar{j} + yz\bar{k}$ taken over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$ (5)
- 27 Evaluate the surface integral $\iint_{\sigma} z^2 ds$, where σ is the portion of the curve $z = \sqrt{x^2 + y^2}$ between $z = 1$ and $z = 3$ (5)
- 28 Use Green's theorem to find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (5)
