Reg No.:_______________ Name:__________________________

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**

Seventh Semester B.Tech Degree Examination December 2022 (2019 scheme)

**Course Code: EET401**

**Course Name: ADVANCED CONTROL SYSTEMS**

**Max. Marks: 100** **Duration: 3 Hours**

## PART A

***Answer all questions, each carries3 marks.*** Marks

1 What do you mean by (*i*) state variables (*ii*) phase variables and (*iii*) state space? (3)

2 A series RLC circuit is excited by a voltage source, *v(t)* volts and the output is measured across the resistor. Derive the state model of the electrical system. (3)

3 State and prove any three properties of state transition matrix. (3)

4 The state space representation and transfer function of a system are given below. Find the value of '*m*'. (3)

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -4 & -m\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u \qquad [y]=[2 \quad 2]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

$$\frac{Y(s)}{U(s)}=\frac{2s+2}{(s+1)(s+4)}$$

5 With necessary equations, explain duality principle. (3)

6 Explain the Gilbert's method to test controllability of the$n^{th}$ order system represented by the state model given below. Assume that the eigen values are distinct. (3)

$$\dot{X}=AX+BU \qquad Y=CX+DU$$

7 Write any three characteristics of non-linear systems. (3)

8 Explain the following (*i*) jump resonance (*ii*) stable limit cycle and (*iii*) unstable limit cycle? (3)

9 Define (*i*) singular point (*ii*) vortex point and (*iii*) saddle point. (3)

10 State the condition/s for a scalar function *V(x)* to be (*i*) positive definite (*ii*) negative definite and (*iii*) indefinite. Give one example for each. (3)

## PART B

***Answer any one full question from each module, each carries 14 marks.***

### Module I

11 a) From basics, derive the state model of an armature controlled dc motor. (7)

b) Differential equation of the system is given by, (7)

$$\frac{d^3y}{dt^3}+6\frac{d^2y}{dt^2}+11\frac{dy}{dt}+6y+u=0$$

where, $y$ is the output and $u$ is the input. Derive the state model of the system using phase variables.

**OR**

12 a) What is similarity transformation? How similarity transformation is helpful for state space analysis? (5)

b) Obtain the diagonal canonical form of the state model given below. (9)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} u \qquad [y] = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

**Module II**

13 A discrete time system is given by, (14)

$$y(k+2) + 5y(k+1) + 6y(k) = u(k)$$

where, $y$ is the output and $u$is the input. Assume zero initial conditions and the sampling period as 1 second. (*i*) Derive the state model of the system in Jordan canonical form (*ii*) Draw the block diagram of the state model and (*iii*) Compute the state transition matrix.

**OR**

14 a) State equation of a system is given below. Compute the state transition matrix using Laplace transform method. Also, compute the solution of the given state equation if the initial state vector is $X(0)$. (7)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

b) Find the state transition matrix using Cayley-Hamilton theorem for the system matrix given below. (7)

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

**Module III**

15 a) Using Kalman's method, check the observability of the system given below. (6)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \qquad [y] = [40 \quad 10 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

b) Design the observer feed back gain matrix for the desired eigen values at -4, -3±j1 for the state model of the system given below. (8)

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}1 & 2 & 0\\ 3 & -1 & 1\\ 0 & 2 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}2\\ 1\\ 1\end{bmatrix}u \qquad [y]=\begin{bmatrix}0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

**OR**

16 a) Using PBH test, check the controllability of the system given below. (6)

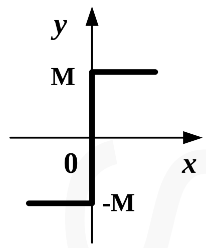
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -4 & -5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u \qquad [y]=\begin{bmatrix}2 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

b) Design a feed back controller with state feed back so that the closed loop poles are at -2, -1±j1. The state equation of the original systems is given below. (8)

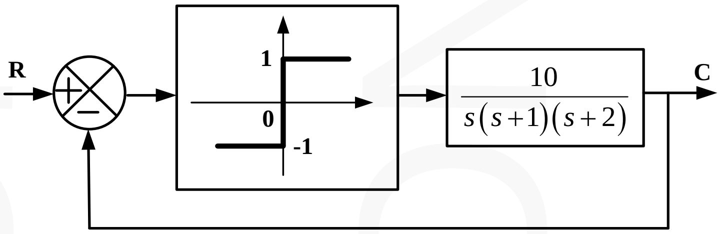
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 10\end{bmatrix}u$$

## Module IV

17 a) Derive the describing function of ideal relay non-linearity given below. (6)

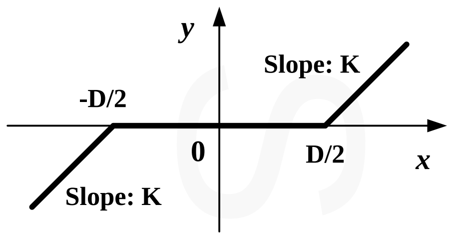



b) Determine the frequency and nature of the limit cycle for the unity feed back system given below. (8)

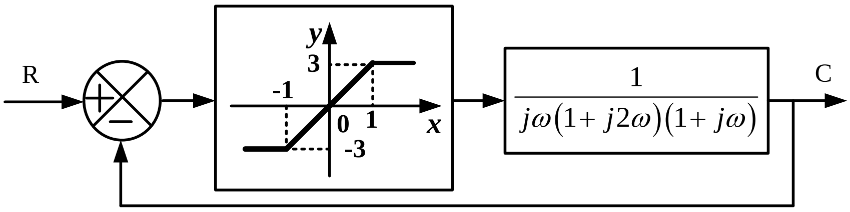



**OR**

18 a) Derive the describing function of dead-zone non-linearity given below. (6)




b) Determine the frequency and nature of the limit cycle for the unity feed back system given below. (8)




## Module V

19 a) What do you mean by phase trajectory? Explain how to draw the phase trajectory using isocline method. (7)

b) Compute the Lyapunov function, *V(X)* for which the system given below is asymptotically stable. (7)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

**OR**

20 a) Construct the phase trajectory of the second order system given below using isocline method. (7)

$$\dot{x}_2 + (2\xi\omega_n)x_2 + \left(\omega_n{}^2\right)x_1 = 0$$

$$X(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix} \qquad \xi = 0.15 \qquad \omega_n = 1\text{rad/s}$$

where *x1* and *x2* are the state variables.

b) Explain Lyapunov function? Using Lyapunov function, *V(X)*, state the condition/s for the system to be (*i*) stable (*ii*) asymptotically stable in the large and (*iii*) unstable. (7)

Consider the system described by,

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_1 - x_2^3$$

Check the stability of the system for the Lyapunov function, $V(X) = x_1^2 + x_2^2$.

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